



# R in Hydrology Workshop: GEOSTATISTICAL PREDICTION OF STREAMFLOW INDICES: TOP-KRIGING via the R-package rtop

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# Overview

## WHY?

- Geostatistical interpolation of streamflow indices and its advantages over conventional approaches to statistical regionalization (special focus on Topological Kriging, or Top-kriging)

## WHAT?

- Top-kriging interpolation in a nutshell
- Real-world applications (examples)

## HOW?

- Tutorial: Top-kriging in R with R-package '*rtop*'

# Introduction: PUB – Prediction in Ungauged Basins

*Decade on PUB, promoted by IAHS (Sivapalan et al., HSJ, 2003)*

In operational hydrology, the scientific community has addressed since the '70's the PUB problem by systematically applying hydrological classification as a tool through regionalization studies.

## Example: Regional Flood Frequency Analysis

Standard and up-to-date approaches to RFFA are described in detail in the textbook by Hosking and Wallis (1997) and in the Flood Estimation Handbook (FEH, 1999)



**Regionalization**  
(e.g. Hosking & Wallis, 1997)

- A. Identification of homogeneous regions
- B. Choice of a frequency distribution
- C. Estimation of the frequency distribution
- D. Validation of the regional model

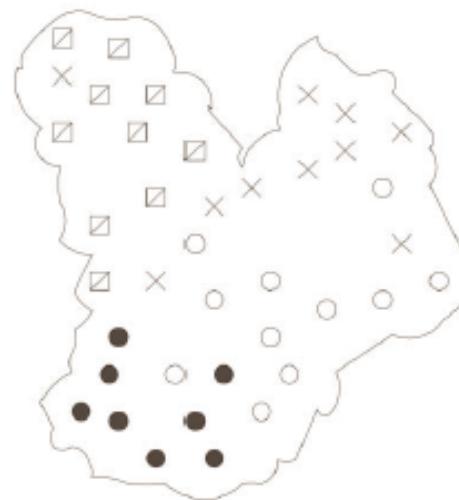
# Introduction: evolution of homogenous regions

Evolution of the concept of hydrologically homogeneous region  
(from Ouarda et al., *J. Hydrol.*, 2001)



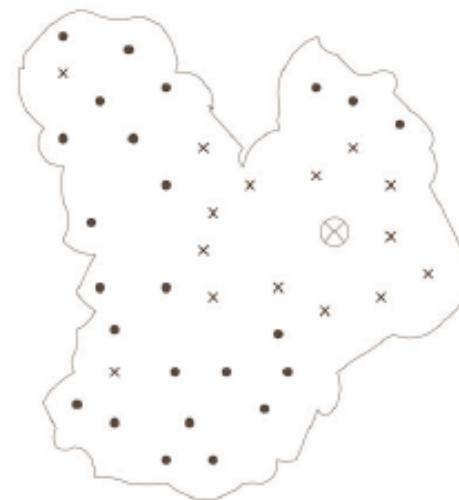
- region 1
- region 2
- region 3
- region 4

a) geographically continuous regions



- region 1
- region 2
- region 3
- region 4

b) non-contiguous homogeneous regions

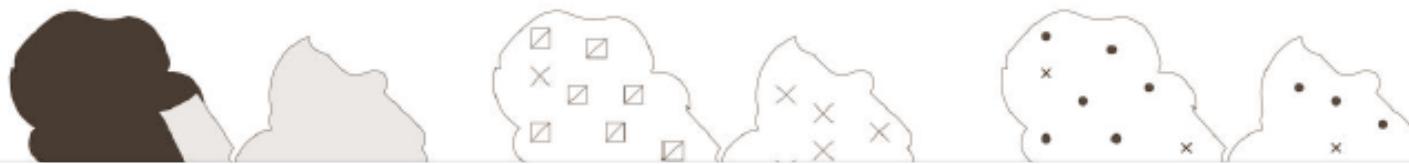


- ⊗ ungauged target site
- × neighboring station
- non-neighboring station

c) hydrologic neighborhoods

# Introduction: evolution of homogenous regions

Evolution of the concept of hydrologically homogeneous region  
(from Ouarda et al., *J. Hydrol.*, 2001)



Several studies highlight that geostatistical interpolation can be effectively applied to the problem of regionalization of hydrometric information:

*Canonical Kriging or Physiographical-Space Based Interpolation PSBI*  
(e.g. Chockmani and Ouarda, *WRR*, 2004; Castiglioni et al., *J. Hydrol.*, 2009).

*Top-kriging*

(e.g. Skøien et al., *HESS*, 2006; Skøien and Blöschl, *WRR*, 2007).



a) geographically  
continuous regions

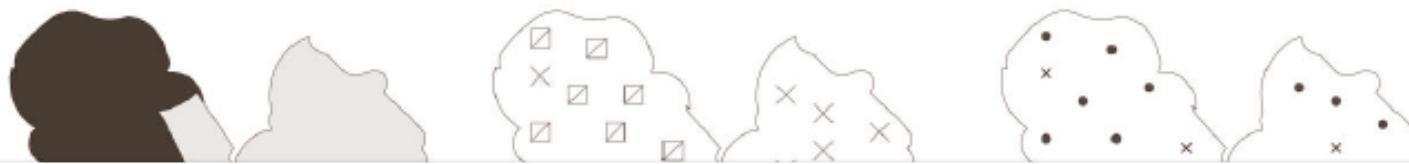


b) non-contiguous  
homogeneous regions

c) hydrologic  
neighborhoods

# Introduction: evolution of homogenous regions

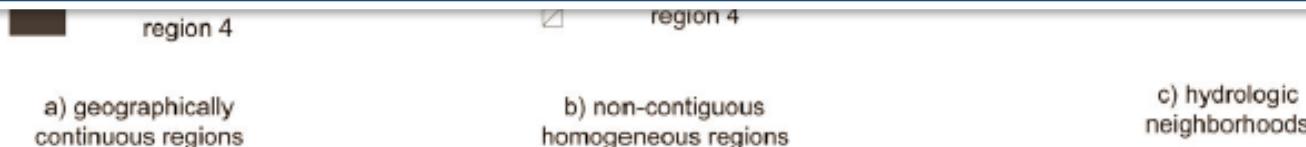
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***Top-kriging* (TODAY'S FOCUS)**  
**(e.g. Skøien et al., *HESS*, 2006; Skøien and Blöschl, *WRR*, 2007).**



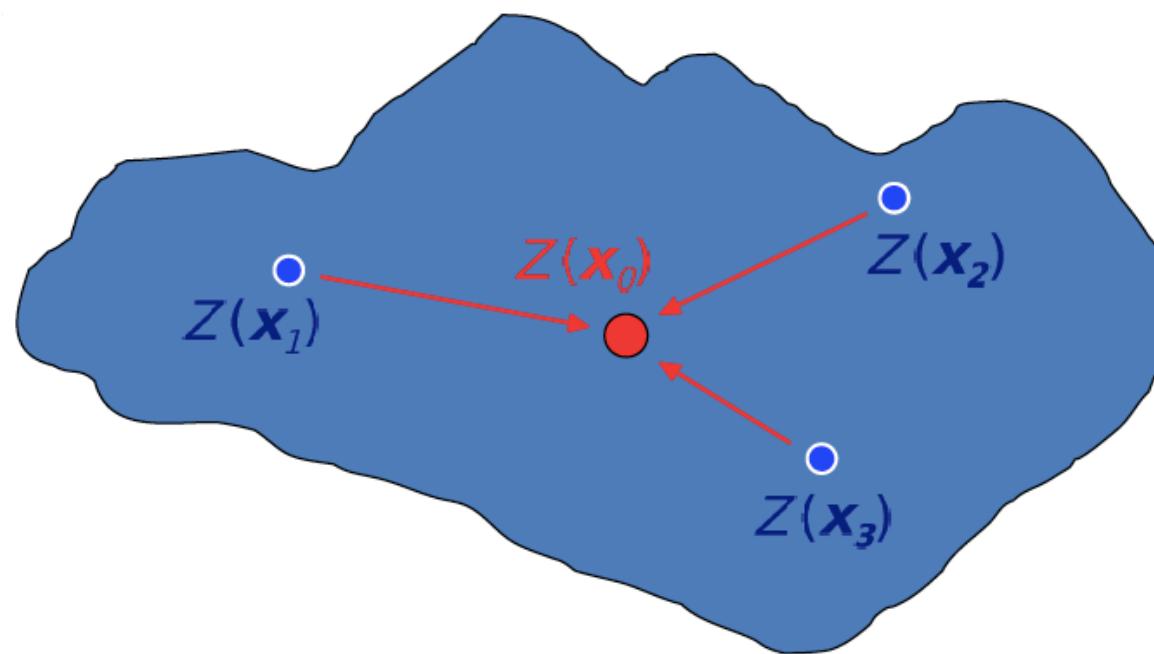
# From Kriging to Top-kriging

## Kriging as a linear estimator

$$\hat{Z}_0 = f(Z_1, Z_2, \dots, Z_n)$$

/ function of observed data

estimated value  
for  $Z_0$

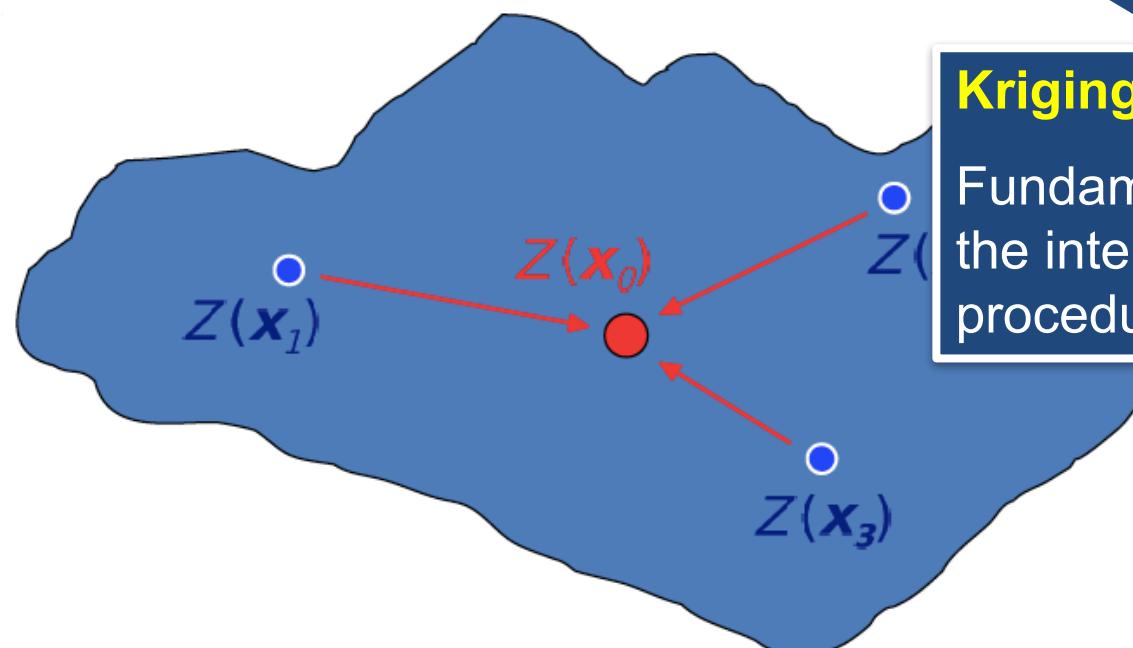


# From Kriging to Top-kriging

## Kriging as a linear estimator

$$\hat{Z}_0 = f(Z_1, Z_2, \dots, Z_n)$$

$$\hat{Z}_0 = \lambda_{0,1} \cdot Z_1 + \lambda_{0,2} \cdot Z_2 + \dots + \lambda_{0,n} \cdot Z_n = \sum_{i=1}^n \lambda_{0,i} Z_i$$



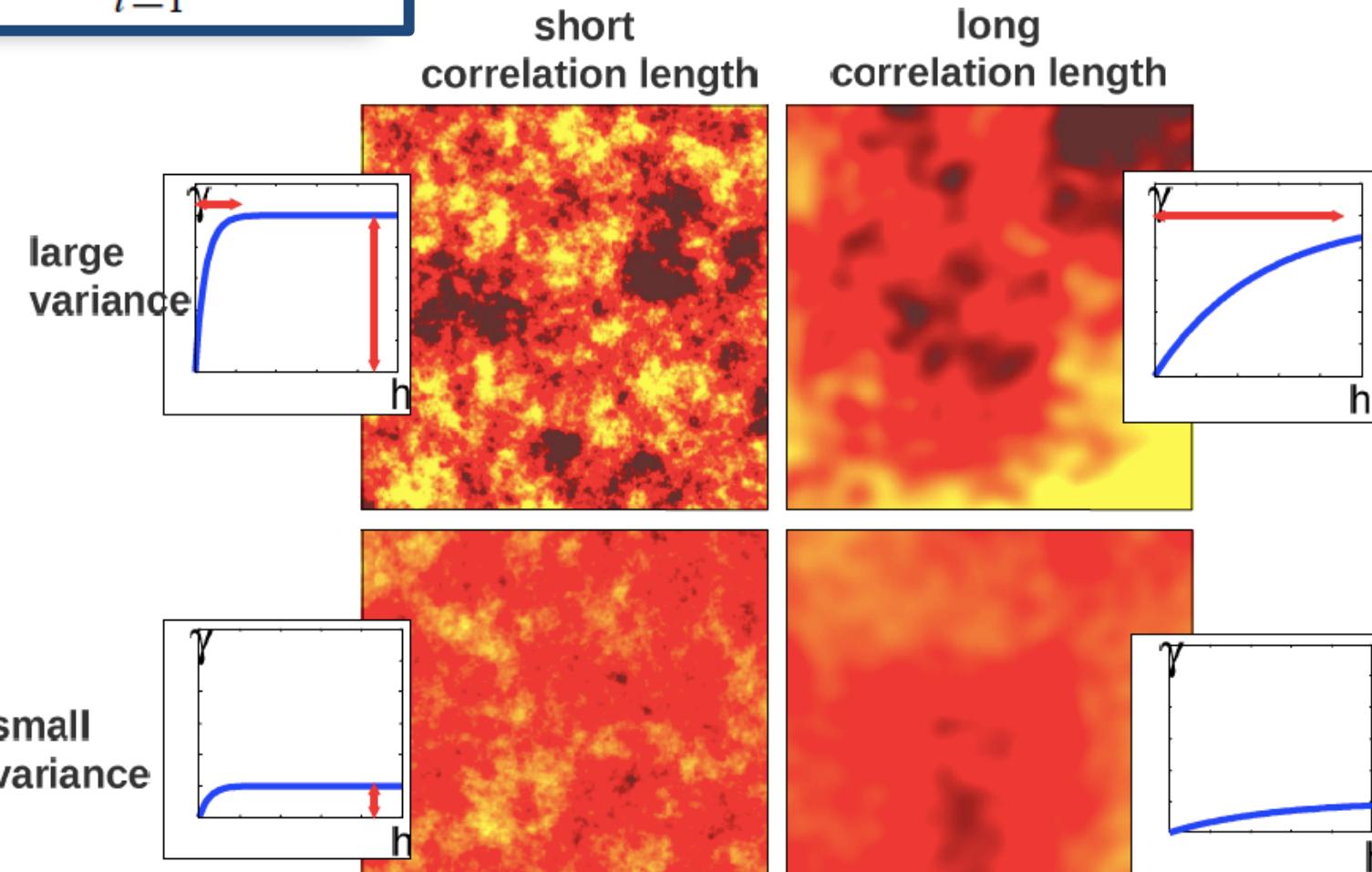
### Kriging weights

Fundamental role in the interpolation procedure

# From Kriging to Top-kriging

$$\hat{Z}(x_0) = \sum_{i=1}^N \lambda_i Z(x_i)$$

Weights from the “Variogram” (graphical representation of variance in space)



# From Kriging to Top-kriging

## Kriging as a BLUe

**Best**

$$E[(Z_0 - \hat{Z}_0)^2] \rightarrow \text{minimize}$$

minimum  
**square error**

**Linear**

$$\hat{Z}_0 = \sum_{i=1}^n \lambda_{0,i} \cdot Z_i$$

**weighted**  
average

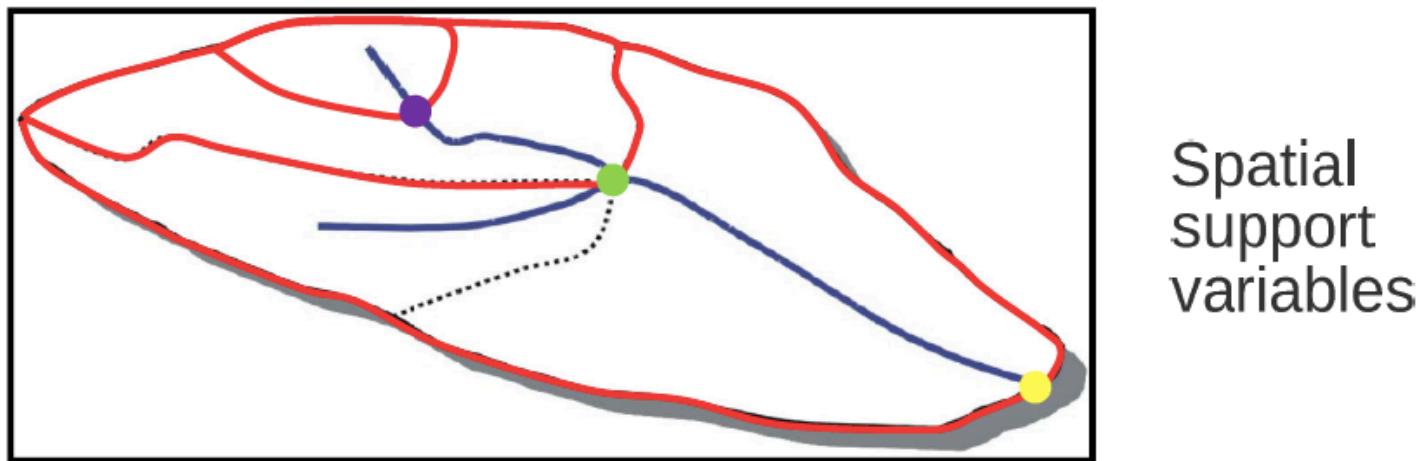
**Unbiased**

$$E[Z_0 - \hat{Z}_0] = 0$$

no systematic  
**error (bias)**

# From Kriging to Top-kriging

**Geostatistics on stream networks:** block-kriging using the drainage basin as support area



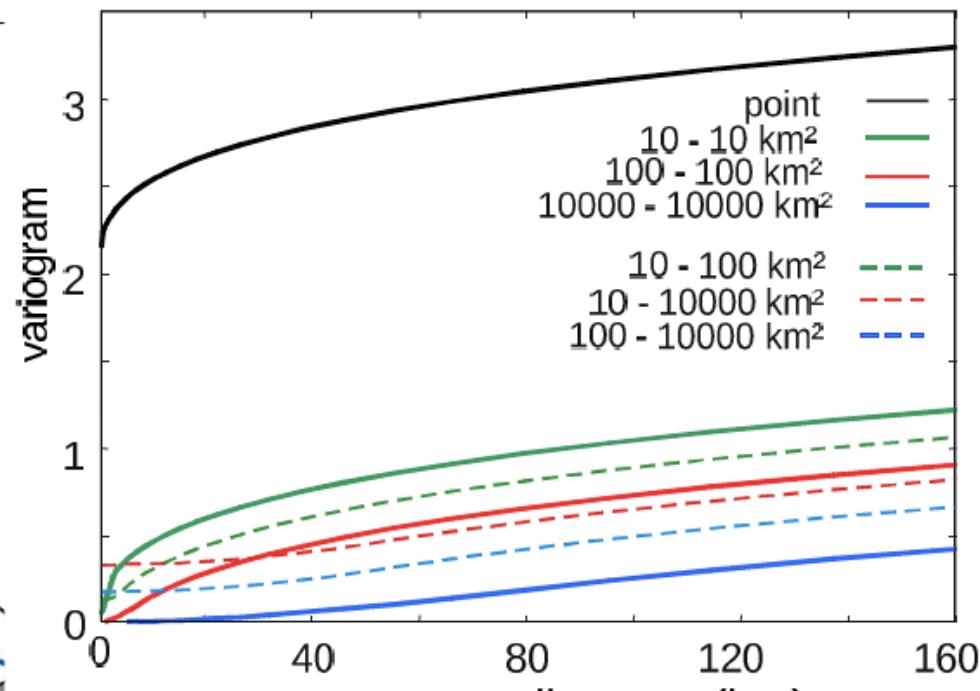
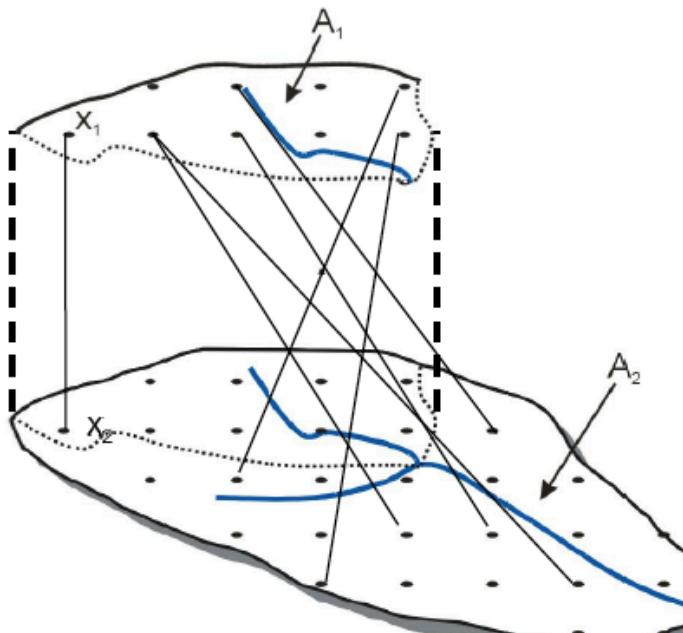
- Standard geostatistics assume random variability
- Topological kriging to account for nested catchment topology

# From Kriging to Top-kriging

**Geostatistics on stream networks: block-kriging using the drainage basin as support area**

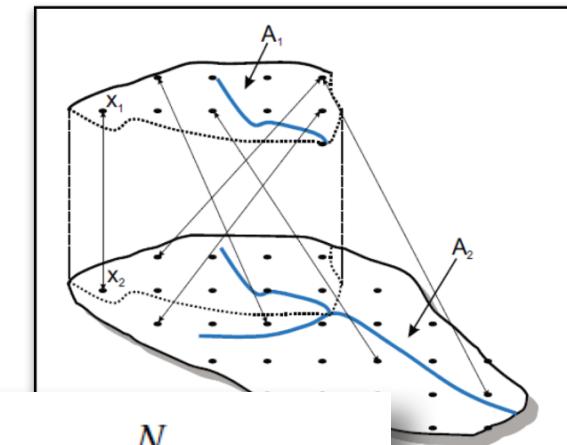
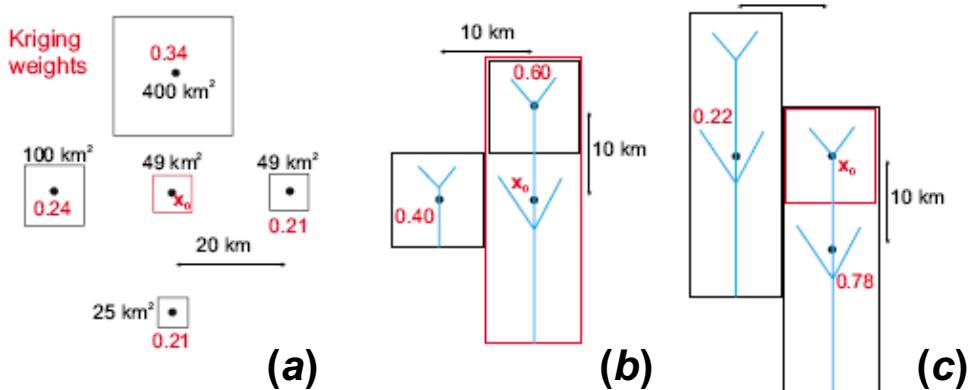
$$\gamma_{12} = 0.5 * \text{Var}(z(A_1) - z(A_2)) = \frac{1}{A_1 A_2} \int \int_{A_1 A_2} \gamma_p(|x_1 - x_2|) dx_1 dx_2 -$$

$$0.5 * \left[ \frac{1}{A_1^2} \int \int_{A_1 A_1} \gamma_p(|x_1 - x_2|) dx_1 dx_2 + \frac{1}{A_2^2} \int \int_{A_2 A_2} \gamma_p(|x_1 - x_2|) dx_1 dx_2 \right]$$



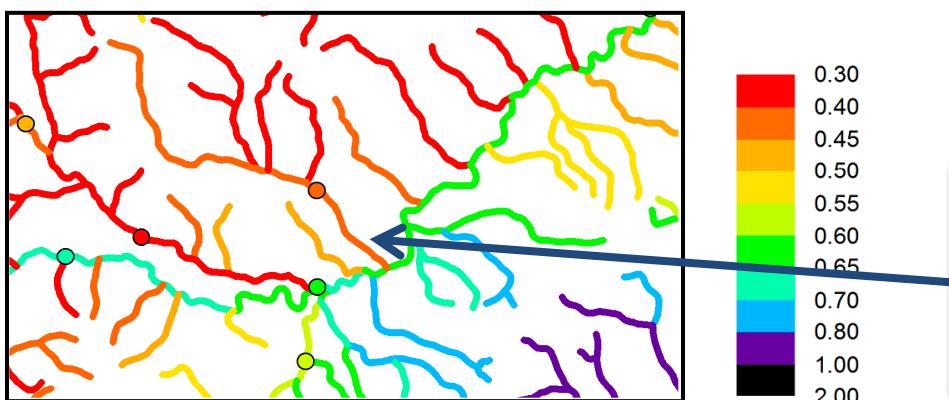
# Top-kriging

Topological kriging, Top-kriging, predicts the variable of interest along river networks taking both the area and nested nature of catchments into account.



$$\hat{Z}(x_0) = \sum_{i=1}^N \lambda_i Z(x_i)$$

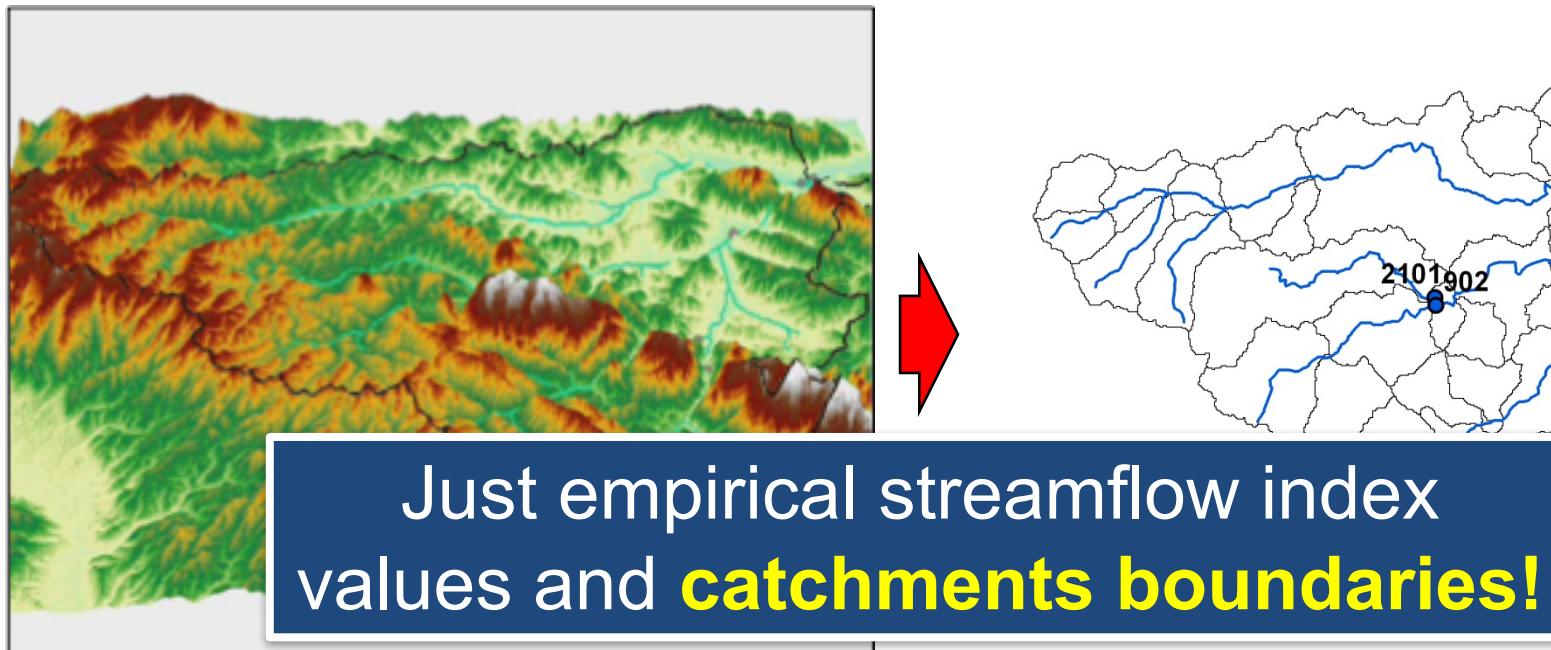
Example of catchment size effect (a) and the effect of nesting (b) and (c) in the estimation of  $\lambda_i$



Applications of Top-kriging:  
Specific 100-year flood ( $\text{m}^3/\text{s}/\text{km}^2$ )  
**Example** (Skøien et al., HESS, 2006)

# Top-kriging: input data for standard applications

Top-kriging application requires the preliminary identification of the stream network and **watershed delineation** for both **gauged catchments and ungauged catchments** for which the streamflow index is to be predicted



DEM, SRTM 90m Digital Elevation Data (<http://csi.cgiar.org/index.asp>)



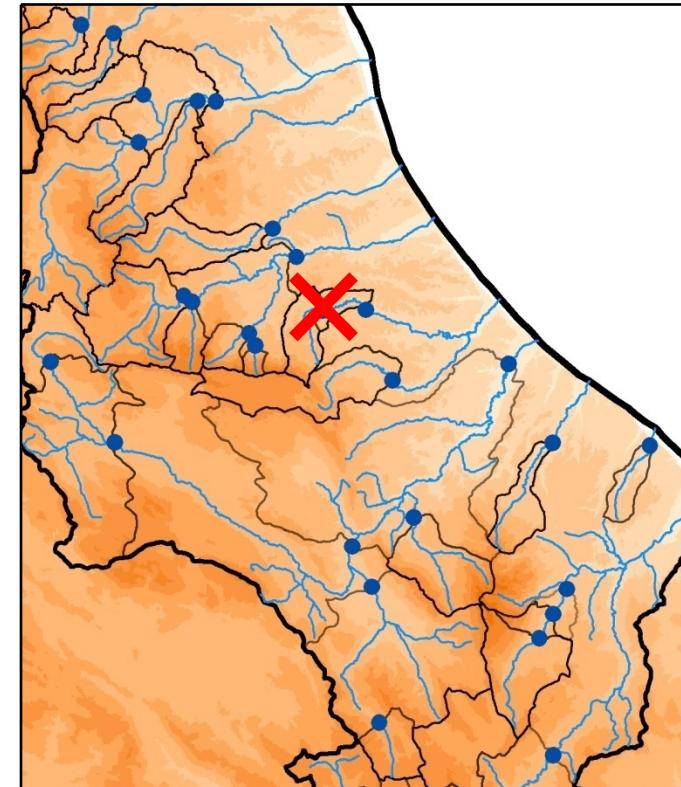
Sub-catchment boundaries ( $A_{min} > 10 \text{ Km}^2$ )

# Leave-One-Out Cross Validation (LOOCV)

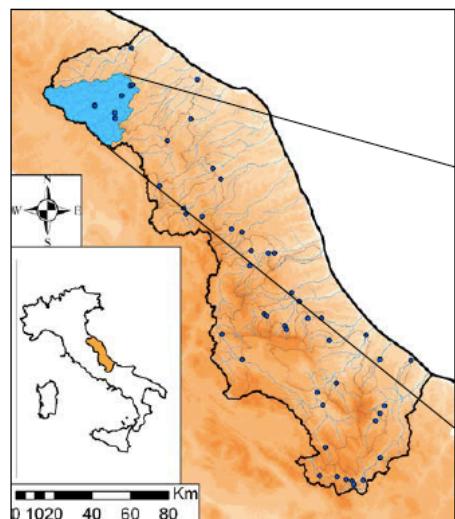
## alternative: Leave-Some-Out Cross Validation

Accuracy of regional predictions in ungauged basins is normally addressed by applying leave-one-out (or leave-some-out) cross-validation procedures (see e.g., Zhang and Kroll, J. Hydrol., 2007):

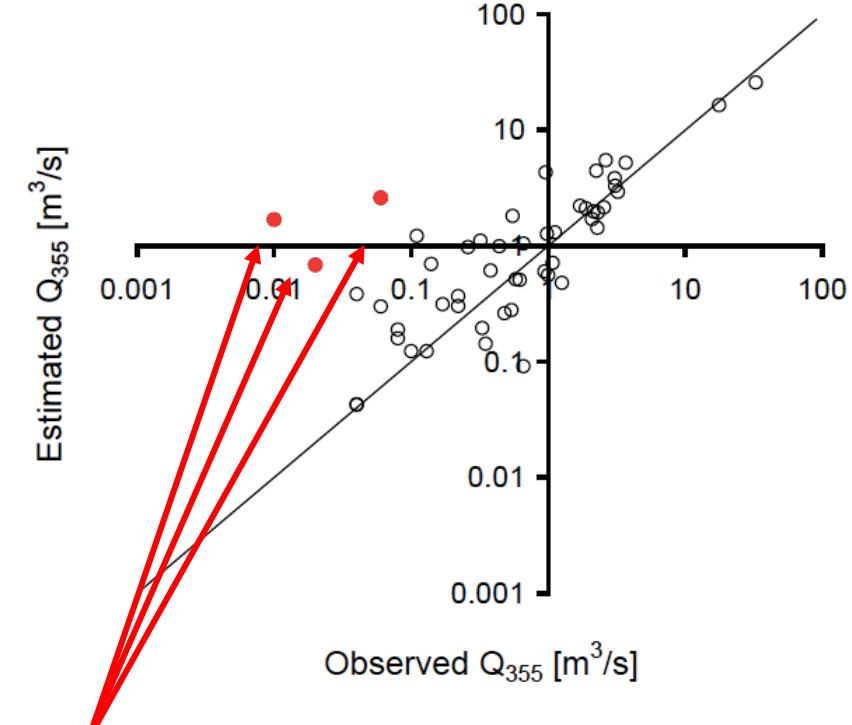
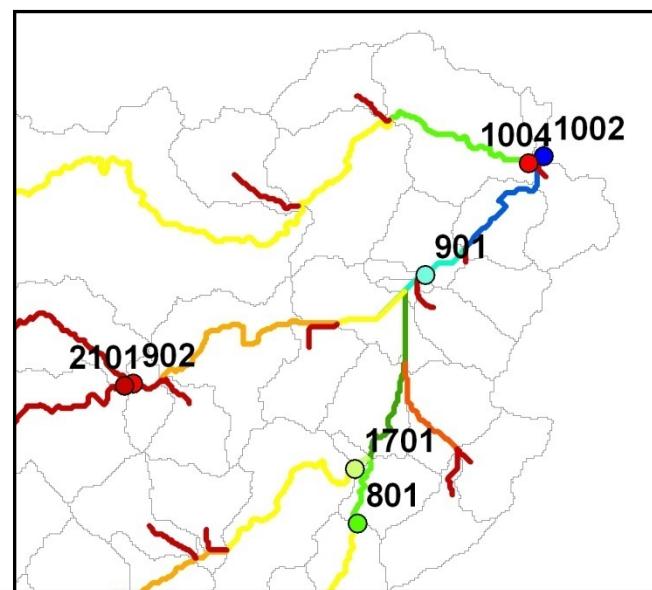
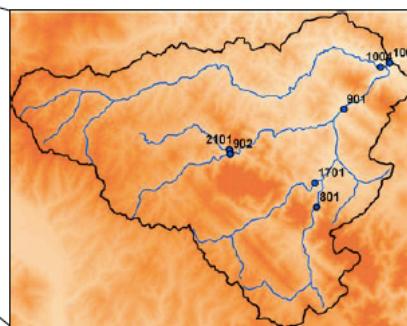
1. assume one of the  $N$  basins, let us say ***site  $i$ , to be ungauged***;
2. predict the ***Q-index*** value for this site on the basis of the ***remaining  $N-1$  observations***;
3. **repeat this step  $N$  times**, considering in turn each of the basins as ungauged, to obtain  $N$  cross-validation estimates of ***Q-index*** which can be compared with the corresponding observations.



# EXAMPLES 1/5: predicting low flows indices

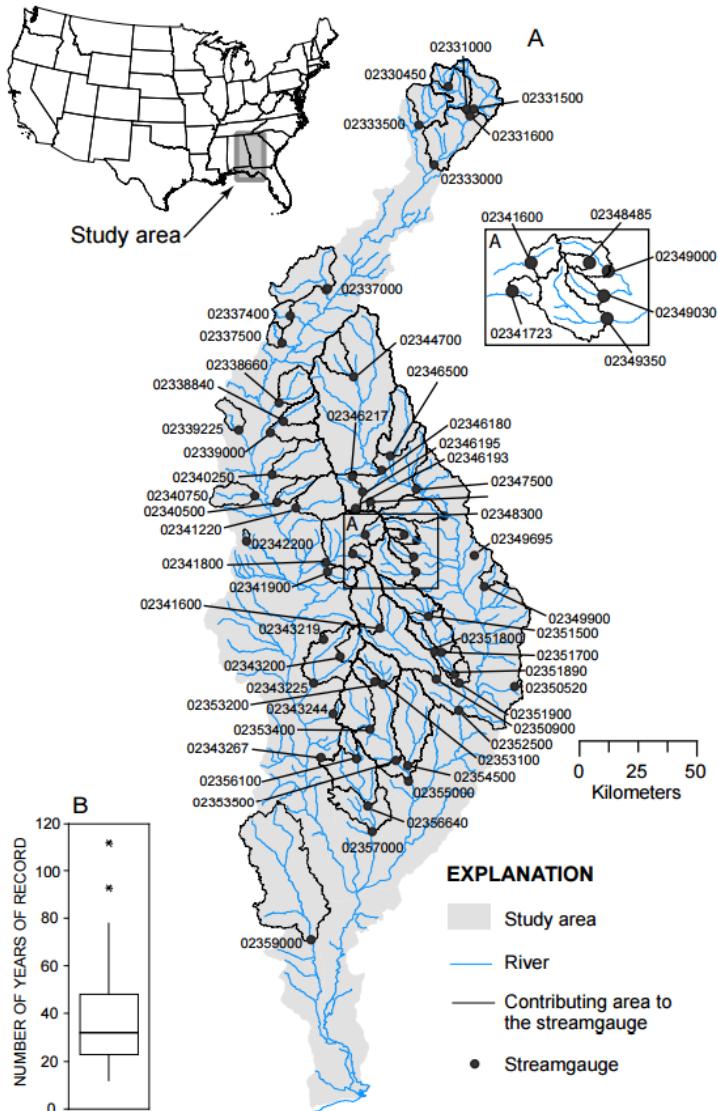


Castiglioni et al. (Hydrol. Earth Syst. Sci., 2011)



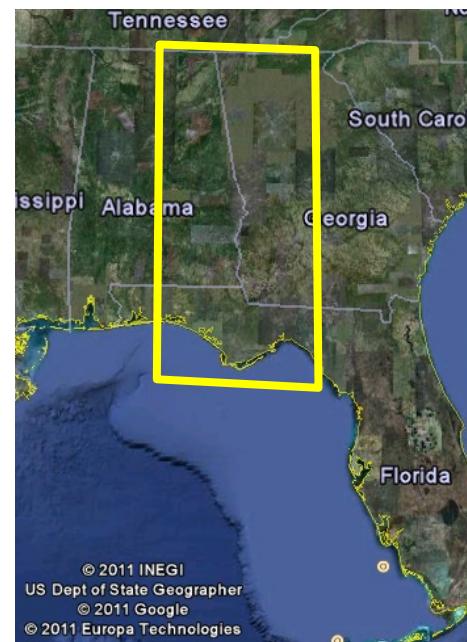
**Red-dots:** limited accuracy for atypical catchments (low permeability and mean elevations, located in peripheral areas), associated with very low low-flow index

## EXAMPLES 2/5: Predicting flood quantiles



Archfield et al. (HESS, 2013) present a comparison of Top-kriging and PSBI performances, and a possible way to combine the approaches

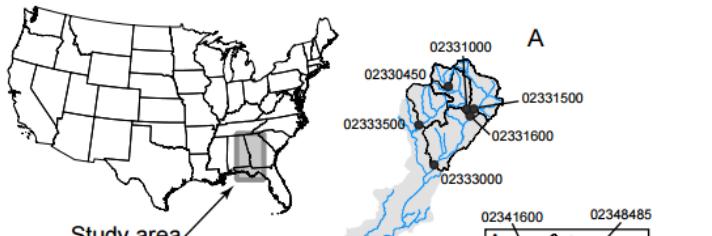
**Study area:** 61 gages located across the southeastern US.



From USGS:

- T-year flood estimates
- Several physiographic and climatic descriptors
- Benchmark prediction procedure (GLS)

# EXAMPLES 2/5: Predicting flood quantiles

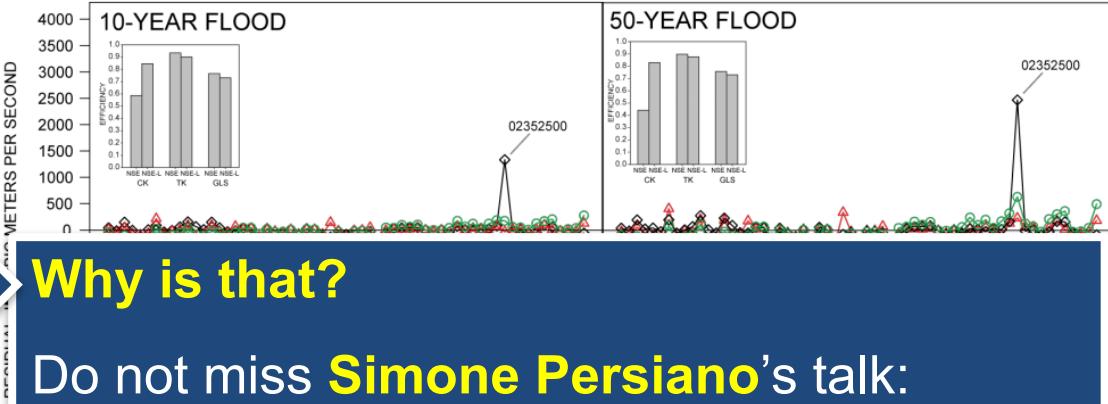


## RESULTS:

For flood quantiles (climate is a strong driver), **Top-KRIGING** **SISTEMATICALLY** outperforms other regionalization techniques, such as the benchmark multi-regression model (GLS model)



Archfield et al. (HESS, 2013) present a comparison of Top-kriging and PSBI performances, and a possible way to combine the approaches.



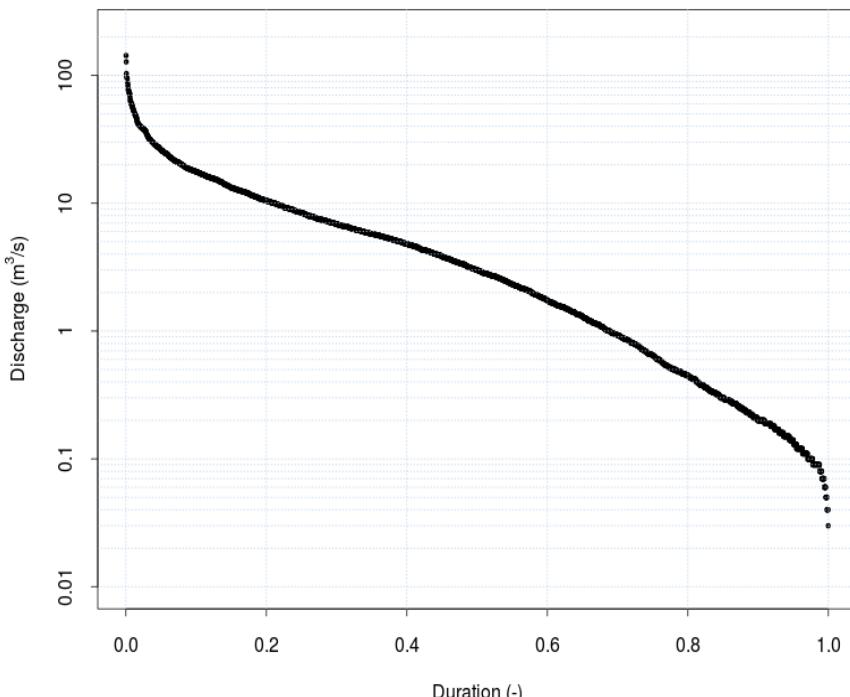
## Why is that?

Do not miss **Simone Persiano**'s talk: IUGG19-1776: The Value of Spatial Correlation in Regional Flood Frequency Analysis: Exploring the Potential of Generalized Least Squares and Top-kriging

**Thursday, 11:15 – 11:30, room 519B**  
**H16c - Floods: Processes, Forecasts, Probabilities, Impact Assessments and Management**

## EXAMPLE 3/5: Predicting flow-duration curves

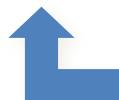
Flow-Duration Curve (FDC)



Represents the percentage of time (duration) in which the streamflow can be exceeded or equalled

Construction of an empirical period of record FDC

1. Sort all available records in descending order (from high to low flows)
2. Choose an appropriate plotting position (e.g. Weibull)
3. Plot each streamflow value against the corresponding ranked duration



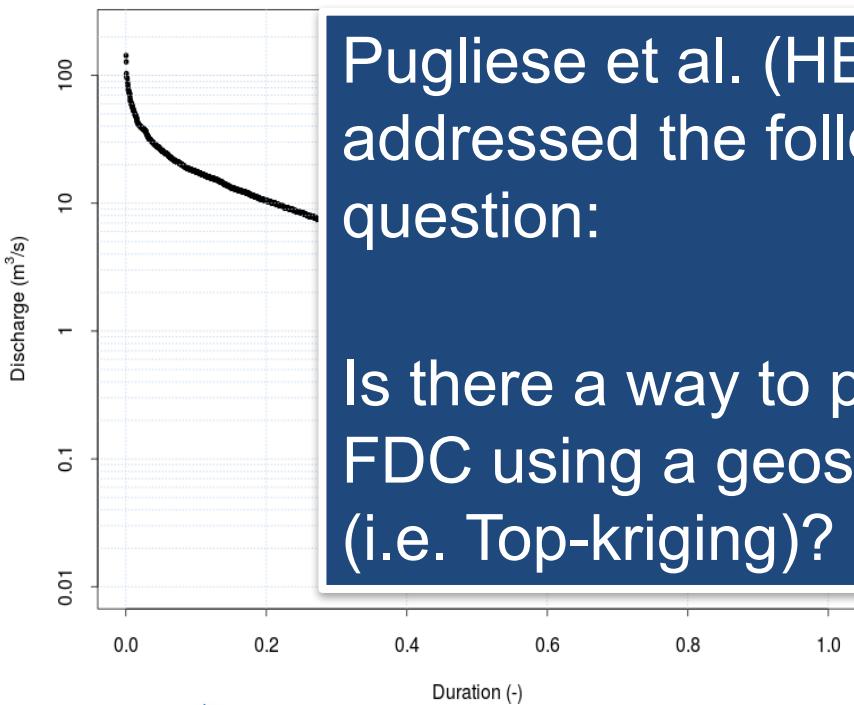
Provides a comprehensive graphical view of the overall streamflow variability for a specific site of interest.

## EXAMPLE 3/5: Predicting flow-duration curves

Flow-Duration Curve (FDC)



Represents the percentage of time (duration) in which the streamflow can be exceeded or equalled

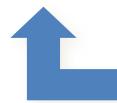


Pugliese et al. (HESS, 2013) addressed the following science question:

Is there a way to predict the entire FDC using a geostatistical technique (i.e. Top-kriging)?

against the corresponding ranked duration

ical period  
cords in  
rom high to  
iate  
(g. Weibull)  
new value



Provides a comprehensive graphical view of the overall streamflow variability for a specific site of interest.

## EXAMPLE 3/5: Predicting flow-duration curves

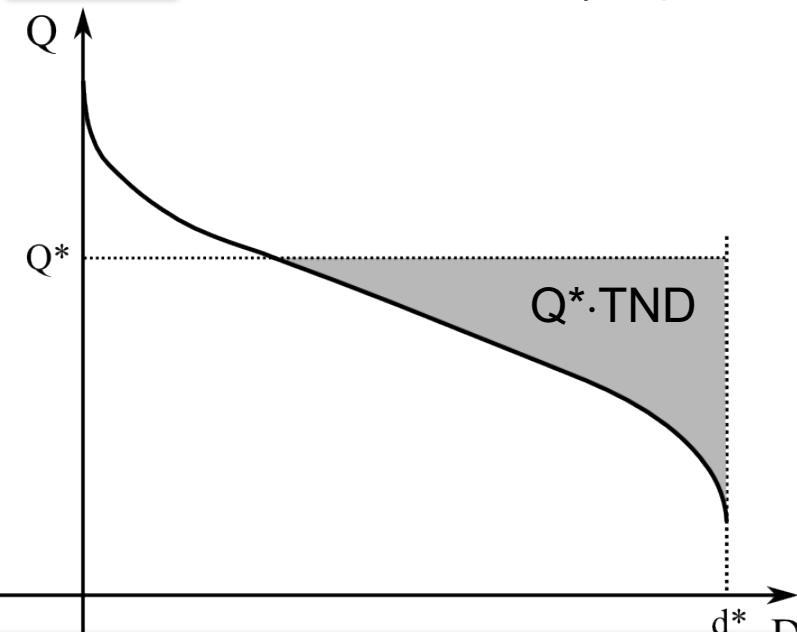
$$FDC = \Psi(x, D)$$



Multidimensional problem



The idea is to summarize the information from the entire curve through a single index (a metric) which identifies its main characteristics (slope, extremes values, mean value, etc...).



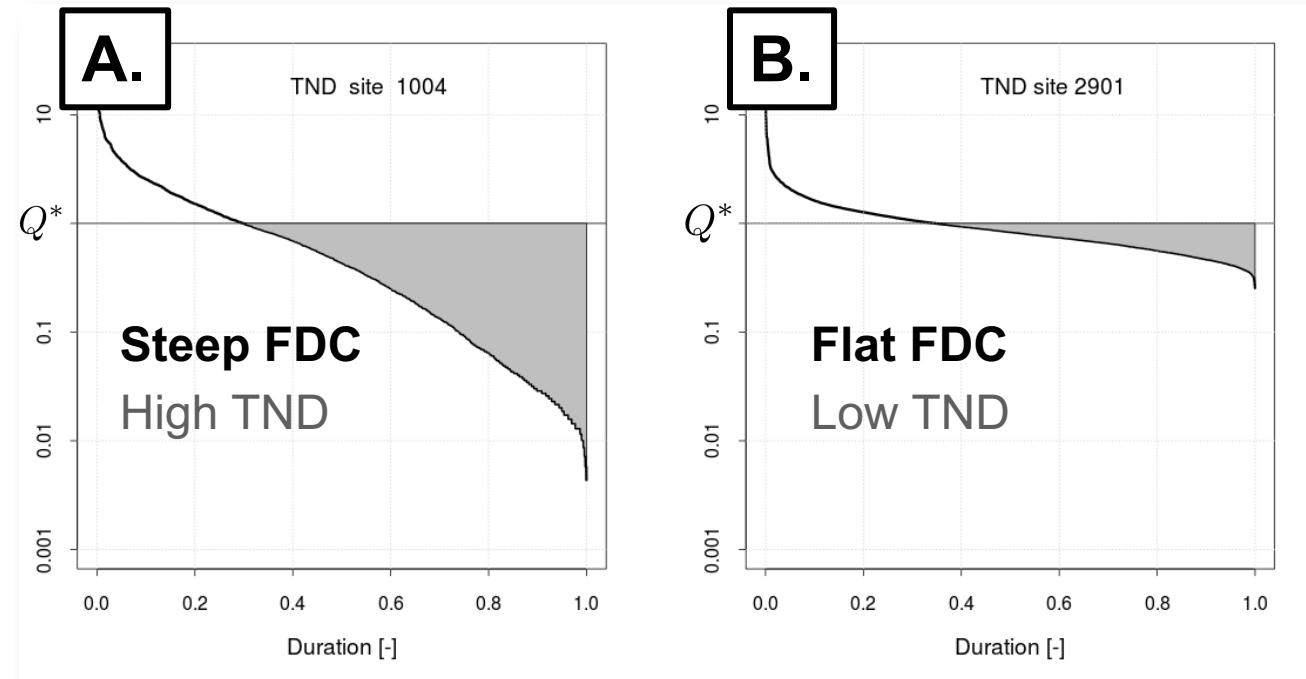
**TND: Total Negative Deviation**

$$TND = \sum_{i=1}^m \left| \frac{Q_i - Q^*}{Q^*} \right| \cdot \Delta D_i = \sum_{i=1}^m |q_i - 1| \cdot \Delta D_i$$

where  $Q^*$  is a reference streamflow (e.g. Mean Annual Flow).

## EXAMPLE 3/5: Predicting flow-duration curves

Why TND? 1) objective; 2) easy to compute; 3) meaningful



- A.** Catchment with rapidly responding runoff
- B.** Catchment with higher storage capacity

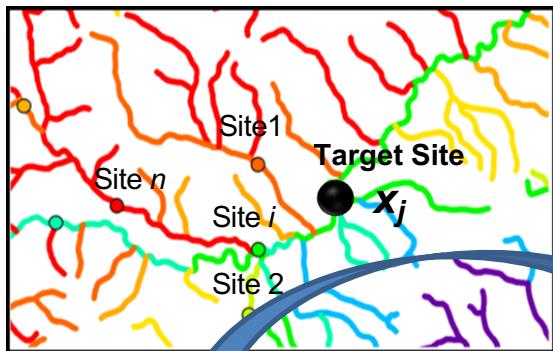
## EXAMPLE 3/5: Predicting flow-duration curves

Using  $Q^*$  as the standardization value, we can combine a traditional **“index-flow” regional method**, with a **geostatistical weighting scheme**:

1. Compute empirical TND values for all  $N_{\text{staz}}$  gauged sites

$$TND_k = \sum_{i=1}^m |q_{i,k} - 1| \Delta_{i,k} \quad k = 1, \dots, N_{\text{staz}}$$

2. Predict TND in the ungauged site  $x_j$  by using Top-kriging (TK).



**INDEX-FLOW**

$$\Psi(x_j, D) = \hat{Q}_j^* \hat{\psi}(x_j, D)$$

Reliable model  
for predicting  $Q^*$

3. Obtain a weighting array

$$\{\lambda_1, \dots, \lambda_n\}$$

**GEOSTATISITCAL WEIGHTING SCHEME**

$$\hat{\psi}(x_j, D) = \sum_{i=1}^n \lambda_i \psi(x_i, D)$$

## EXAMPLE 3/5: Predicting flow-duration curves

Extensive application:  
USGS-SEMC Dataset

### Dataset

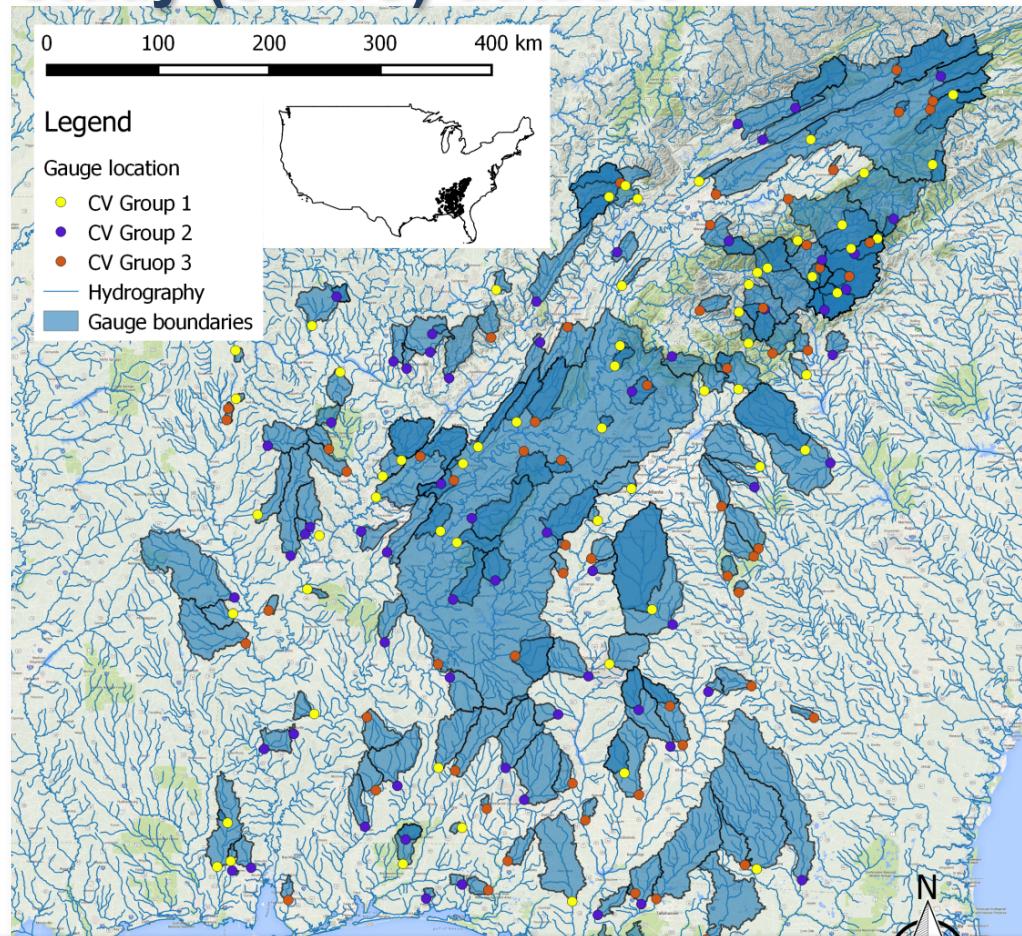
- 182 river catchments
- 30 yrs. observed streamflows
- Several geomorphological and climatic catchment descriptors
- USGS-SEMC FDCs prediction for benchmarking (quantile regression method, 27 quantiles resampling, 3-fold cross-validation **3FCV**)

### Information used by TNDFK

- Watershed boundaries (SHP files)
- Streamflow data

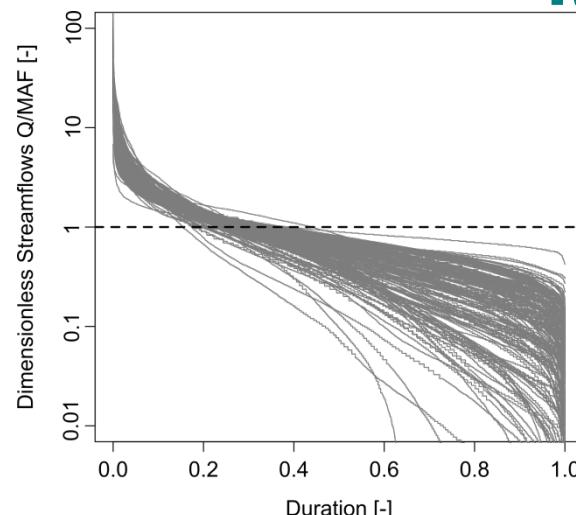
(a **3FCV** may have a significant impact on a geostatistical procedure...)

### Southeast Model-Comparison study (SEMC) dataset

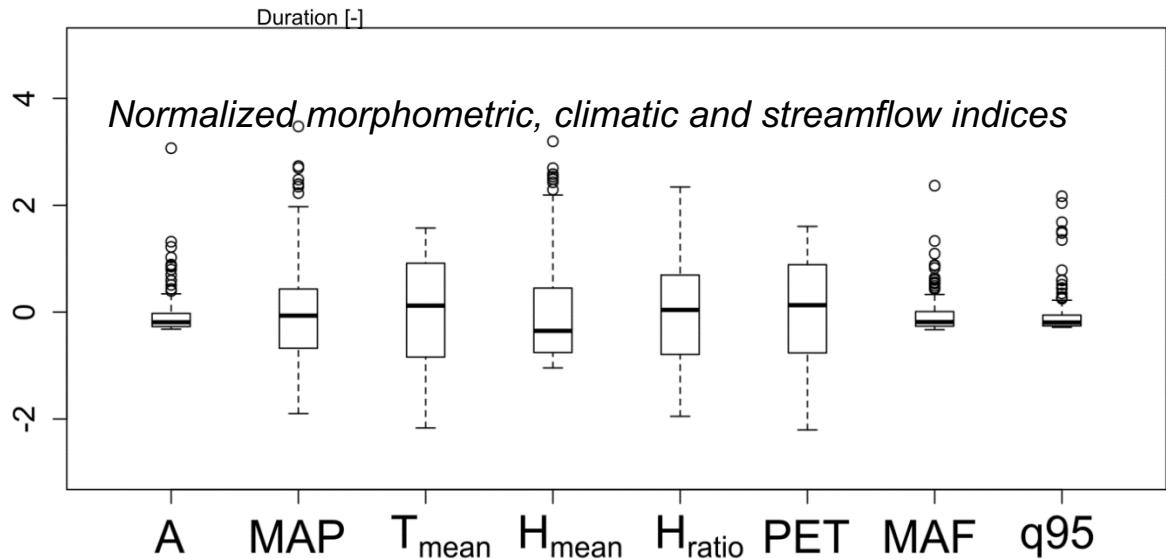


# USGS-SEMC Dataset:

## Main characteristics



	A (km <sup>2</sup> )	MAP (mm)	T <sub>mean</sub> (°C)	H <sub>mean</sub> (m)	H <sub>ratio</sub> (-)	PET (mm/d)	MAF (m <sup>3</sup> /s)	q95 (m <sup>3</sup> /s)
Min.	15.3	1146	9.5	17.7	0.2	576.5	0.3	0
1 <sup>st</sup> Qu.	225.9	1358	13.1	116.0	0.3	753.5	3.3	0.05
Median	588.4	1462	15.7	252.1	0.4	862.2	6.9	0.12
Mean	1427.2	1473	15.4	370.8	0.4	846.1	15.4	0.13
3 <sup>rd</sup> Qu.	1317.2	1547	17.8	520.7	0.5	954.7	15.9	0.21
Max.	56609.5	2072	19.6	1452.4	0.6	1042.3	598.1	0.59

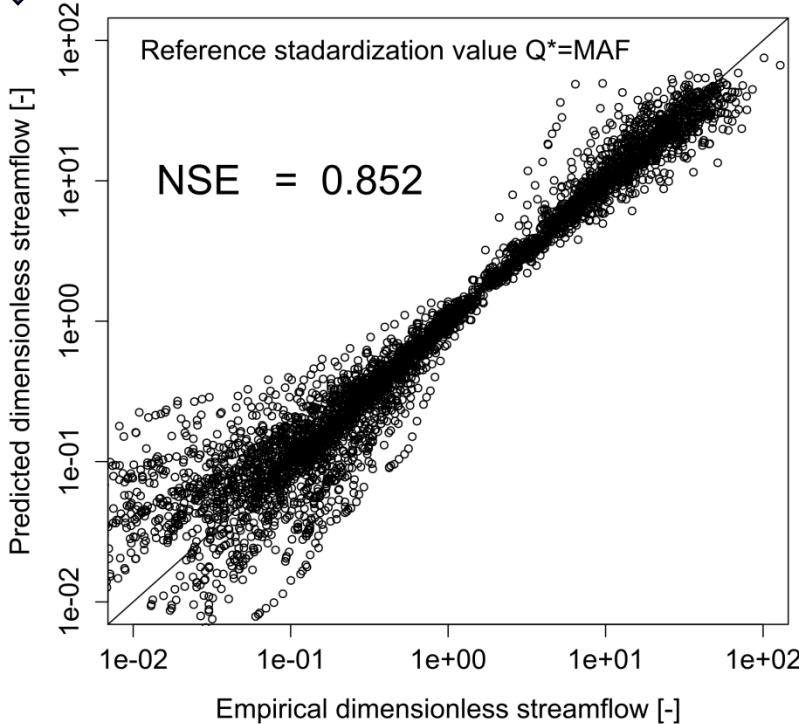


A: Drainage Area  
 MAP: Mean Annual Precipitation  
 T<sub>mean</sub>: Mean Temperature  
 H<sub>mean</sub>: Mean Elevation  
 H<sub>ratio</sub>: Dimensionless elevation relief ratio  
 (difference between mean and minimum elevation divided by the full range of elevations)  
 PET: Potential Evapotranspiration  
 MAF: Mean Annual Flow  
 q95: daily-streamflow exceeded 95% of the time divided by MAF

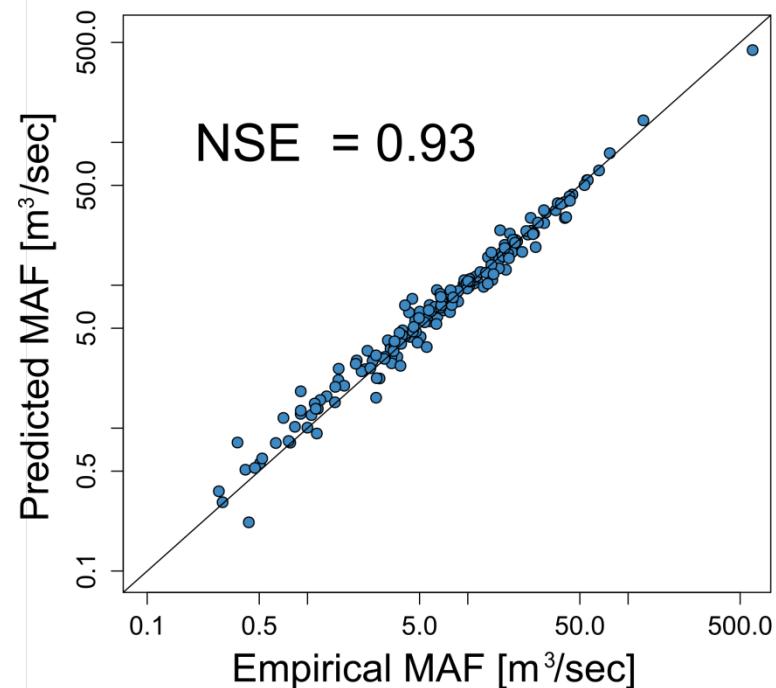
# Results: prediction of standardized FDCs and MAF

**Settings:** (i)  $Q^* = \text{MAF}$ ; (ii) 3-fold cross-validation (3FCV); (iii) 6 donor sites

(1) Prediction of dimensionless FDCs

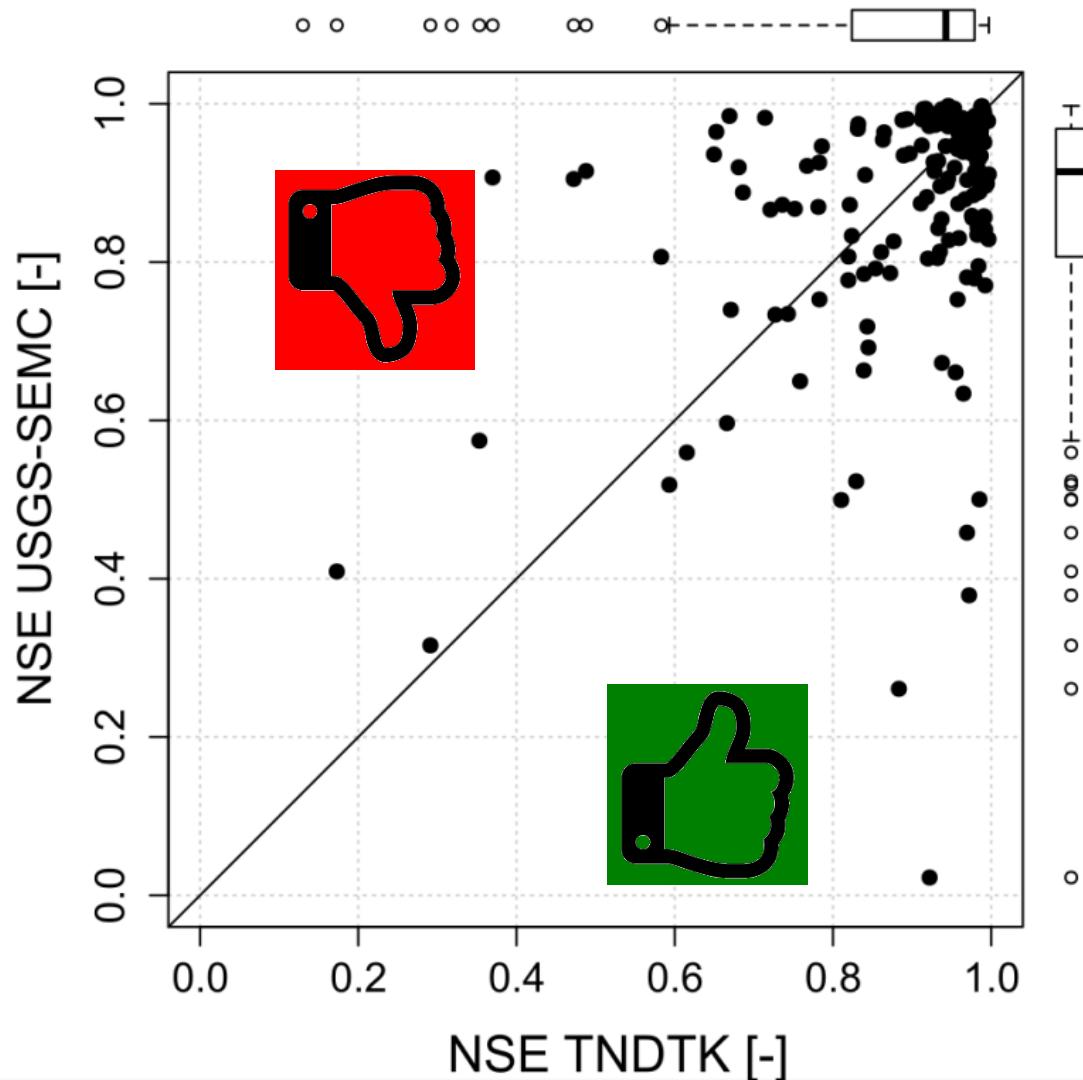


(2) Prediction of index-flow value ( $Q^* = \text{MAF}$ )



## Results: comparison between Top-kriging and USGS-SEMC (site-wise for $n=6$ )

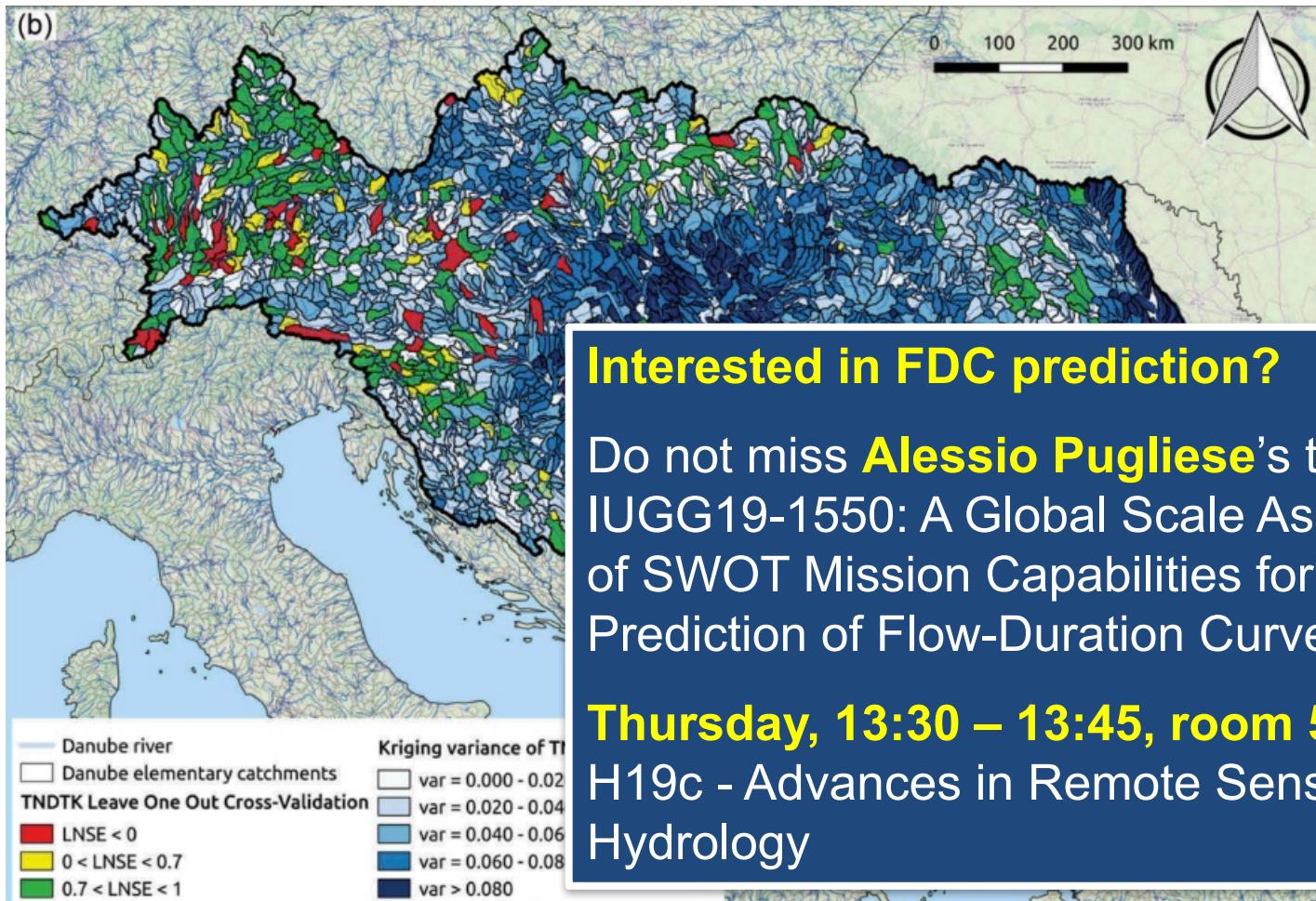
Top-kriging  
outperforms  
USGS-SEMC  
for **102/171**  
(~60%) sites



## Prediction of streamflow regimes over large geographical areas: interpolated flow-duration curves for the Danube region

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<sup>a</sup>Department of Civil, Chemical, Environmental and Materials Engineering (DICAM), School of Civil Engineering, University of Bologna, Bologna, Italy; <sup>b</sup>European Commission, DG Joint Research Centre (JRC), Ispra, Italy

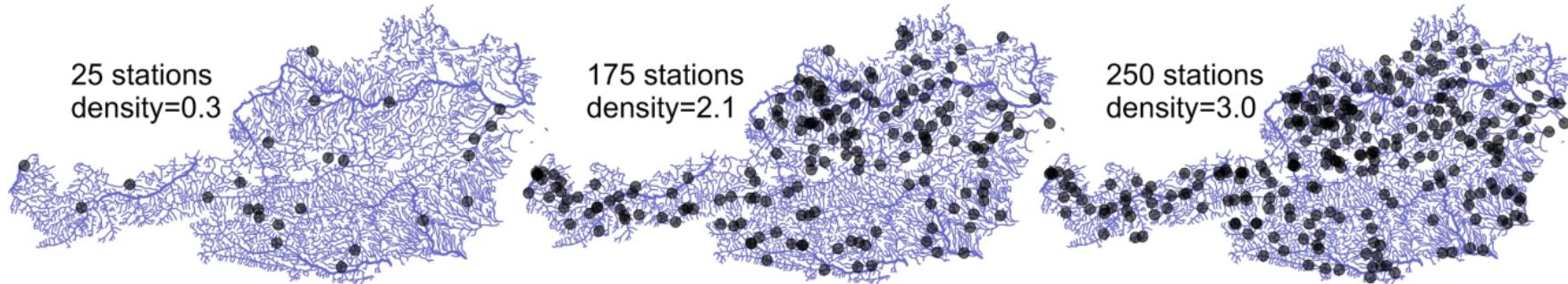


## EXAMPLE 4/5: predicting daily streamflow series

Parajka et al. (J. Hydrol. Hydromech., 2015) investigated the role of station density for predicting daily streamflow series by top-kriging interpolation in Austria (more than 550 stations; i.e.  $>6.5$  stations/1000km $^2$ )

### Results of the cross-validation:

In Austria, top-kriging interpolation is superior to hydrological model regionalisation if station density exceeds approximately 2 stations per 1000 km $^2$  (175 stations in Austria). The average median of Nash-Sutcliffe cross-validation efficiency is larger than 0.7 for densities above 2.4 stations/1000 km $^2$ . For such densities, the variability of runoff efficiency is very small over ten random samples. Lower runoff efficiency is found for low station densities (less than 1 station/1000 km $^2$ ) and in some smaller headwater basins.



**Fig. 2.** Example of 3 different station densities (25, 175 and 250 stations).

# EXAMPLE: interpolation of stream-temperature

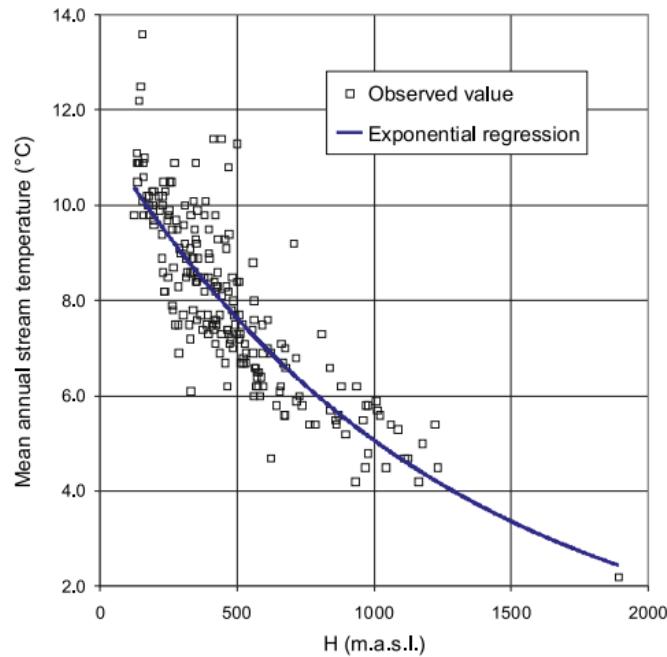
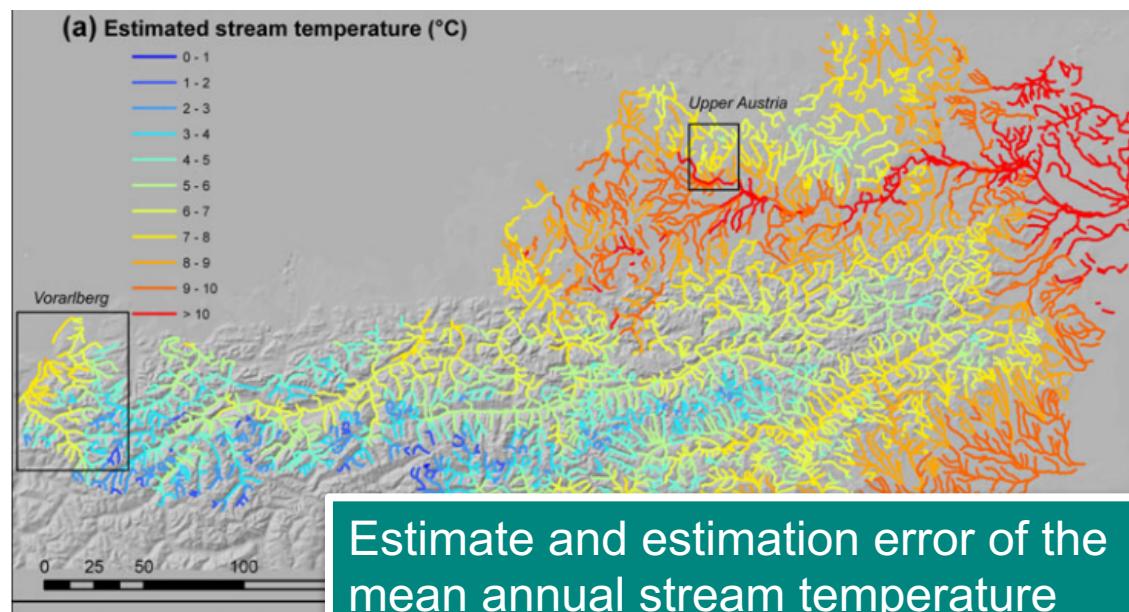
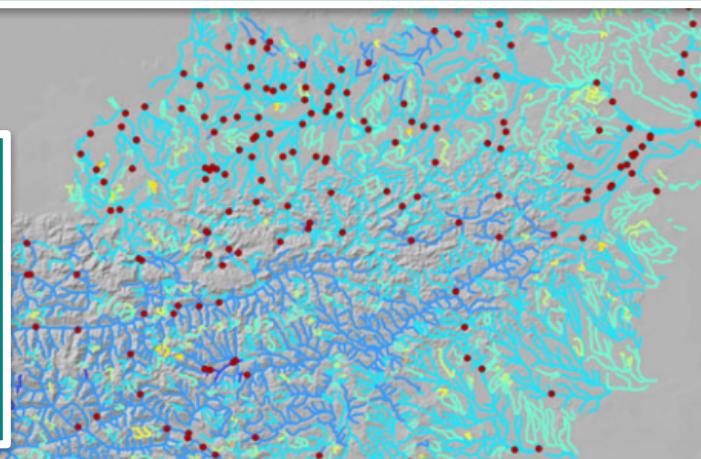


Fig. 4 Exponential regression model between the mean annual stream temperature (in degree Celsius) and altitude of the gauge (in metres above sea level) used as external drift function, fitted to 214 catchments



Estimate and estimation error of the mean annual stream temperature (C°) for Top-kriging with external drift

Laaha et al. (Env. Mod. Assess., 2013) interpolated stream temperatures using top-kriging with an external drift over the whole Austrian stream-network:  
 Exponential regression:  $R^2=0.77$   
 Top-kriging with external drift:  $R^2=0.81$



# Top-kriging applications (incomplete list)

Literature reports successful applications for predicting:

- ✓ Flood quantiles (e.g. Skøien et al., HESS, 2006; Archfield et al., HESS, 2013)
- ✓ Low-flows (Castiglioni et al., HESS, 2011; Laaha et al., HP, 2014)
- ✓ Flow-duration curves (Ad. Water. Resour., 2016; Castellarin et al., HSJ, 2018)
- ✓ Streamflow series (Skøien & Bloeschl, WRR, 2007; Parajka et al., J. Hydrol. Hydromech., 2015)
- ✓ Streamflow temperature (Laaha et al., Env. Mod. Assess., 2013 )
- ✓ Fish-habitat suitability indices (Ceola et al., Adv. Wat. Resour., 2018)
- ✓ ...

**Next application could be yours!  
Have fun with the “rtop” R-package  
(Skøien et al., Comp. & Geosci., 2014)**





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