Uncertainty and Sensitivity Analysis of hydrological models

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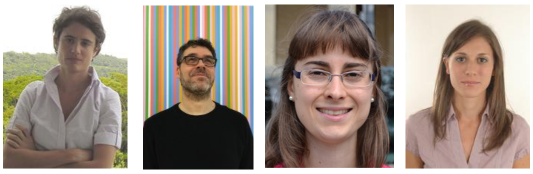

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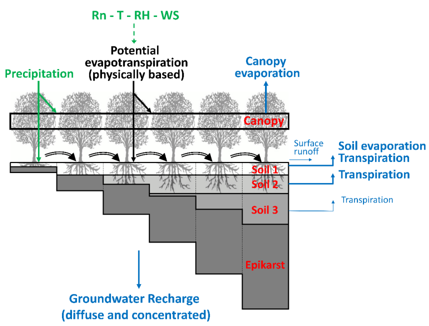
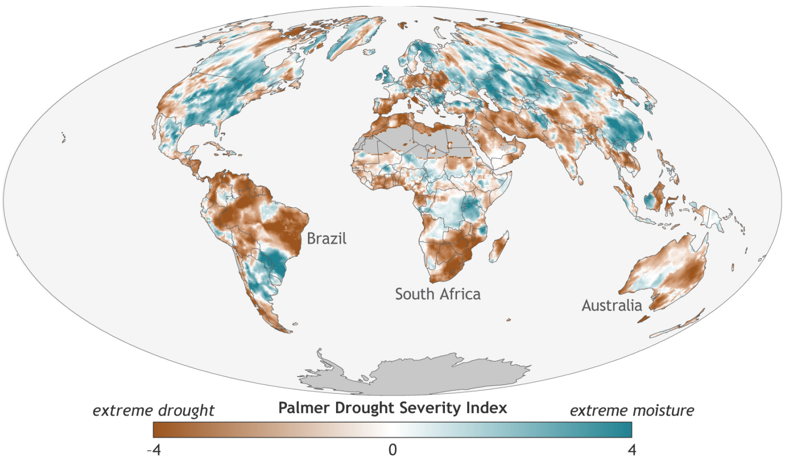
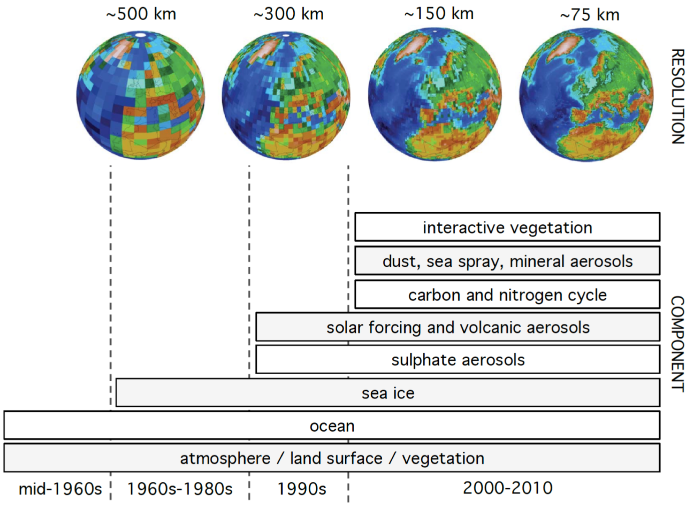
Computer models are essential tools in hydrology to advance our science and inform decision-making





We use increasingly detailed computer models at ever larger scales and finer resolutions



Washington et al *PTRS* 2008

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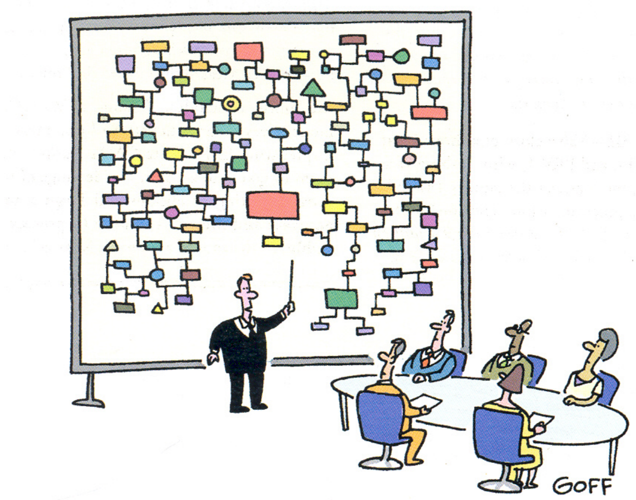
Model complexity is challenging as we quickly lose our ability to understand the model behavior



Does the model provide the "right" answer?

Does it provide the "right" answer for the "right" reason?

What are the priorities for improving the model?

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All modelling studies follow essentially a very similar process

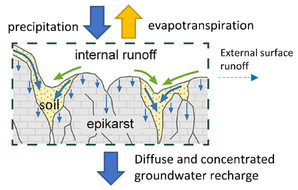
conceptualization
> perceptual model

translation into equations
> mathematical model

implementation into a computer code
> computer model

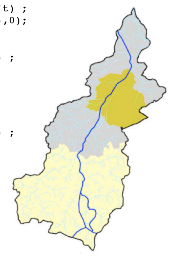
calibration
> computer model tailored to a specific site/system

evaluation/prediction



$$\frac{1}{A}\frac{\partial Q}{\partial t}+\frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+g\frac{\partial y}{\partial x}-g(S_o-S_f)=0$$

```
function Q_sim = hymod_sim(par,prec,evap)
Sm =max(eps,par(1));beta =par(2);alfa=par(3);
Rs=par(4);Rf=par(5); N=length(prec);
for t=1:N
    F  = 1 - (1-sm(t)/Sm)^beta ; Pe(t)  = F*prec(t) ;
    sm_temp = max(min(sm(t) + prec(t) - Pe(t),Sm),0);
    Pe(t)=Pe(t)+max(sm(t)+prec(t)-Pe(t)-Sm,0)+...
          min(sm(t)+prec(t)-Pe(t),0);
    W = min(abs( sm(t)/Sm ),1) ; Ea(t)= W*evap(t) ;
    sm(t+1) = max(min(sm_temp-Ea(t),Sm),0);
    Ea(t)= Ea(t)+ max(sm_temp-Ea(t)-Sm,0)+...
          min(sm_temp-Ea(t),0);
    QsL(t)   = Rs * sL(t) ;
    sL(t+1)  = sL(t) +  (1-alfa)*Pe(t) - QsL(t) ;
    sF1(t+1) = sF1(t) +  alfa*Pe(t) - Rf * sF1(t) ;
    QsF(t)   = Rf * sF1(t) ;
end
Q_sim = QsL+QsF;
```



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All modelling studies follow essentially a very similar process.... which is paved with uncertainties and 'subjective' choices

conceptualization > perceptual model	< uncertain assumptions/epistemic uncertainty
translation into equations > mathematical model	< (some more) uncertain assumptions / computational constraints
implementation into a computer code > computer model	< uncertain choices (+ bugs)
calibration > computer model tailored to a specific site/system	< lack of data *or* uncertain data [both inputs & outputs] → uncertain parameters
evaluation/prediction	< (some more) lack of data *or* uncertainty in data

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Examples of uncertainty in model structure and implementation

HYDROLOGICAL PROCESSES
Hydrol. Process. (2013)
Published online in Wiley Online Library
(wileyonlinelibrary.com) DOI: 10.1002/hyp.10096

Diagnostic evaluation of multiple hypotheses of hydrological behaviour in a limits-of-acceptability framework for 24 UK catchments

G. Coxon,[1]* J. Freer,[1] T. Wagener,[2] N. A. Odoni[1] and M. Clark[3]
[1] *School of Geographical Sciences, University of Bristol, Bristol, UK*
[2] *Department of Civil Engineering, University of Bristol, Bristol, UK*
[3] *Research Applications Laboratory, National Center for Atmospheric Research (NCAR), Boulder, Colorado, USA*

WATER RESOURCES RESEARCH, VOL. 46, W10511, doi:10.1029/2009WR008896, 2010

Ancient numerical daemons of conceptual hydrological modeling: 2. Impact of time stepping schemes on model analysis and prediction

Dmitri Kavetski[1] and Martyn P. Clark[2]

Received 12 November 2009; revised 19 March 2010; accepted 21 April 2010; published 8 October 2010.

[1] Despite the widespread use of conceptual hydrological models in environmental research and operations, they remain frequently implemented using numerically unreliable methods. This paper considers the impact of the time stepping scheme on model analysis (sensitivity analysis, parameter optimization, and Markov chain Monte Carlo-based uncertainty estimation) and prediction. It builds on the companion paper (Clark and Kavetski, 2010), which focused on numerical accuracy, fidelity, and computational efficiency.

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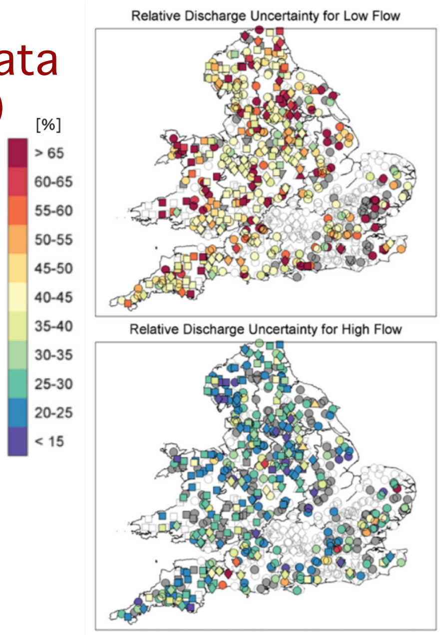
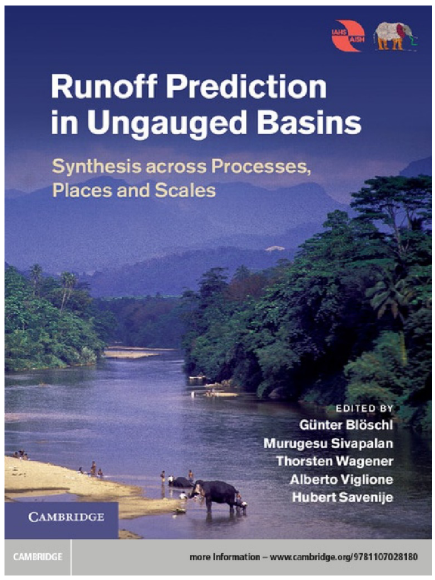
Example of uncertainty in data (or lack of data)





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As a consequence of these multiple and potentially interacting uncertainties ...

The model structure (and/or its numerical implementation) may be inadequate

The model parameters may be poorly estimated or ineffective

The model predictions may be inaccurate for any of the above reasons,
or because the (well identified) model is forced by erroneous input data

... and in either case,

We may over/under-estimate the model's prediction accuracy because of errors in the output observations
or simply not have observations to compare with

So how do we go about constructing, testing ("validating") and using computer models?

We ignore uncertainty and pretend it is not there 🙊🙉🙈

or

[1] we quantify uncertainty in model outputs, so we have an idea of "how wrong/variable" model predictions are given our level of uncertainty/subjectivity in the model set-up → Uncertainty Analysis

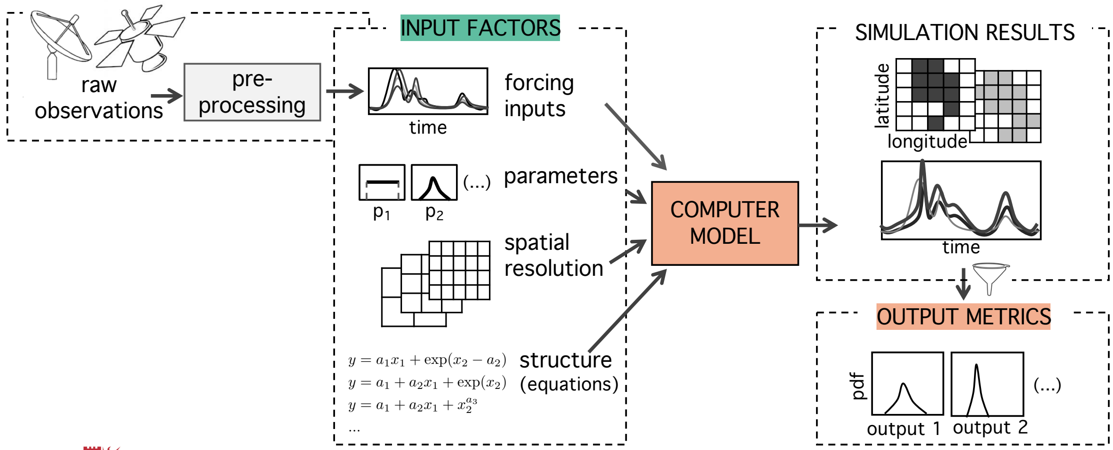
[2] we identify which sources of input uncertainty mostly contribute to output uncertainty, so we know which of them are very critical (and we should tackle first) → Sensitivity Analysis

[1] quantifying uncertainty in model outputs: propagation methods

[1] characterize uncertainty of input factors

[2] propagate uncertainties through the model

[3] summarise and communicate output uncertainty



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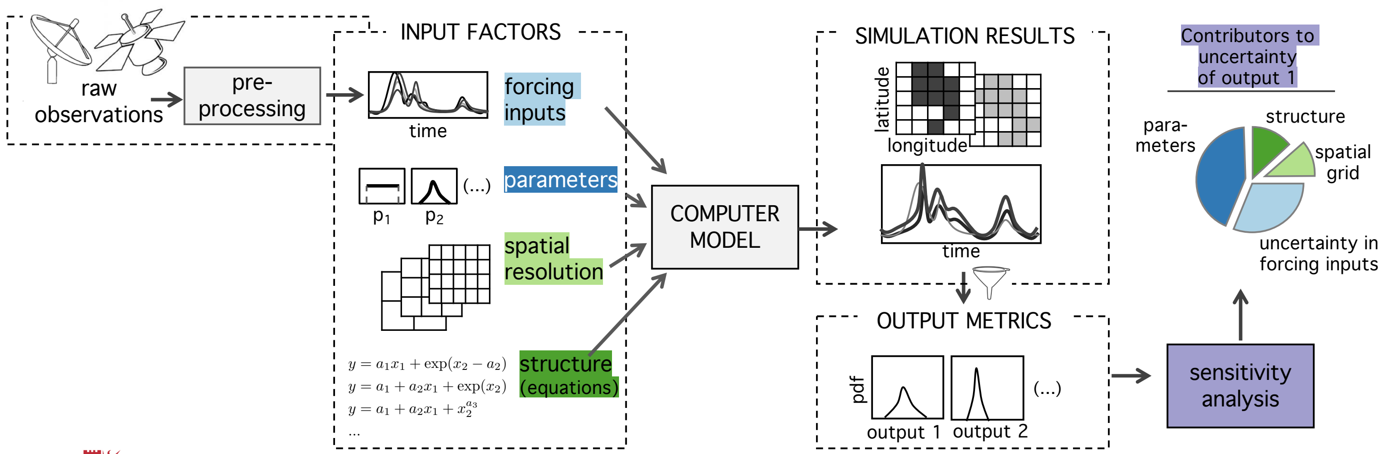
[2] measuring relative contributions to output uncertainty: sensitivity analysis

[1] characterize uncertainty of input factors

[2] propagate uncertainties through the model

[3] summarise and communicate output uncertainty

[4] identify key contributors to output uncertainty



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Aims and scope of this talk

_ discussion of sources of uncertainty
_ Uncertainty Analysis (UA) based on ‘forward-propagation’ (Monte Carlo)
_ a ‘flavour’ of some techniques for (global) Sensitivity Analysis (SA)
_ critical choices in carrying out UA/SA
_ examples of what we can learn from UA/SA

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WHY

doing UA/SA?

EXAMPLE APPLICATIONS

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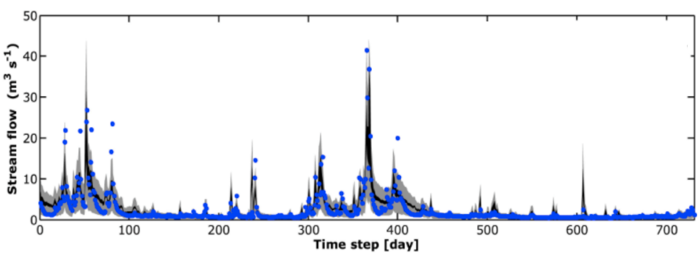
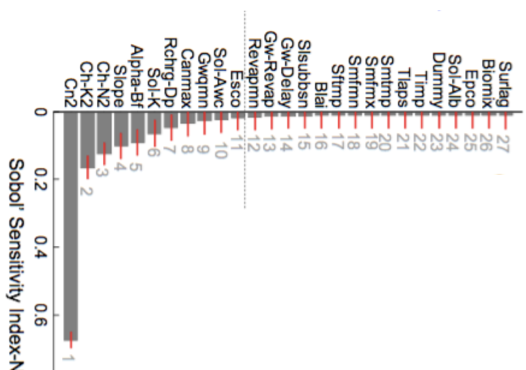
If we have output observations to compare with, which model parameters control the predictions accuracy?

Example application to the SWAT model





Zadeh et al *EMS* 2017

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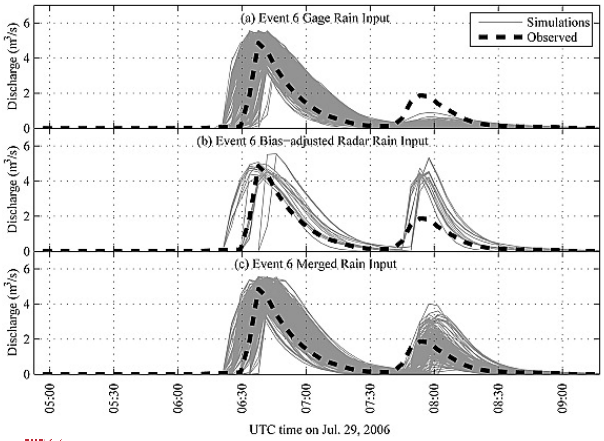
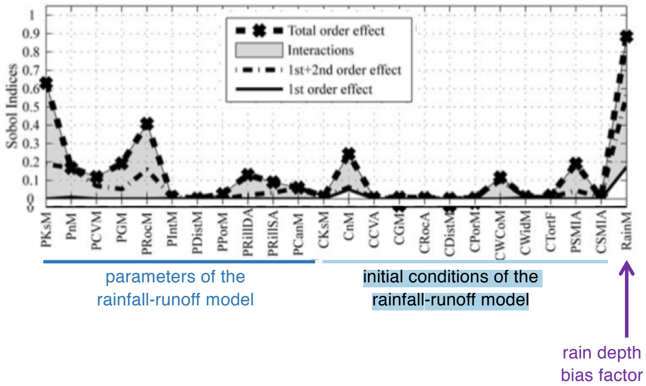
How much is controlled by the model parameters vs parameters of the input data pre-processing?

Application to spatially-distributed rainfall-runoff model for semiarid regions





Yatheendradas et al *WRR* 2008

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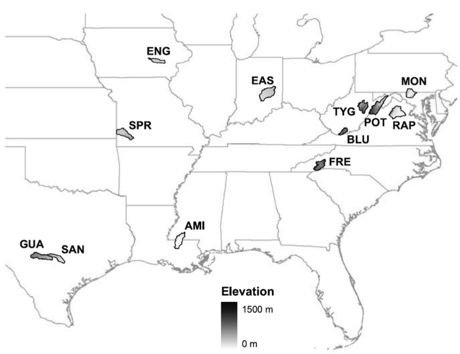
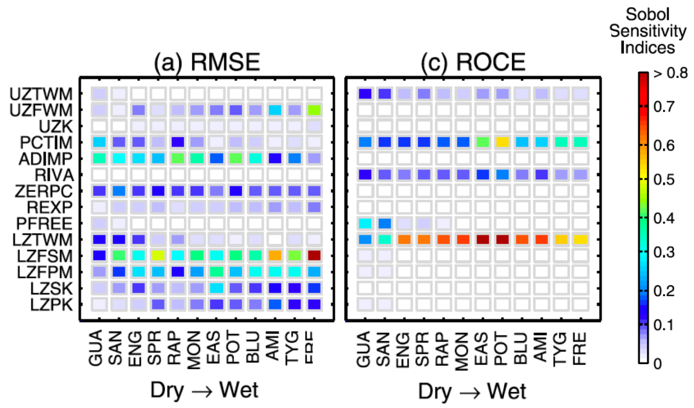
How do dominant controls (parameters) vary across places?

Application to lumped rainfall-runoff model





Van Werkhoven et al *WRR* 2008

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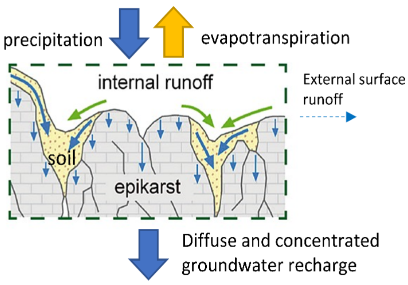
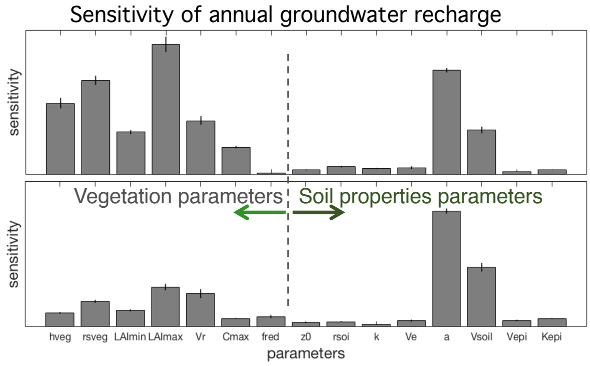
If we do not have output observations, can we at least ensure that the 'right' parameters control the model response?

Application to a karst groundwater recharge model



Sarrazin et al *GMD* 2018



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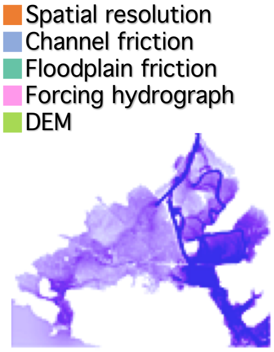
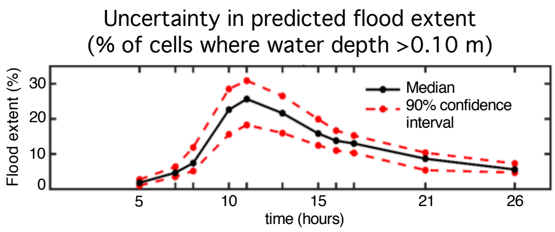
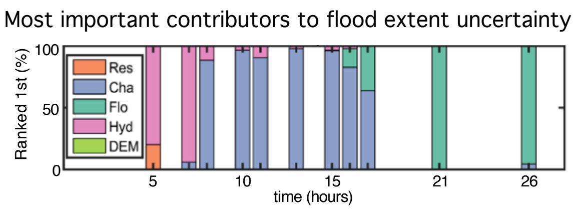
Which other modelling choices also control the model predictions, and when, where and how much?

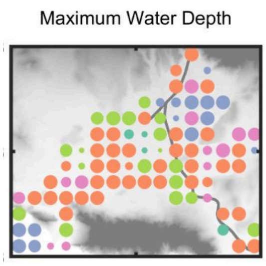
Application to a flood inundation model

Savage et al. *WRR* 2016









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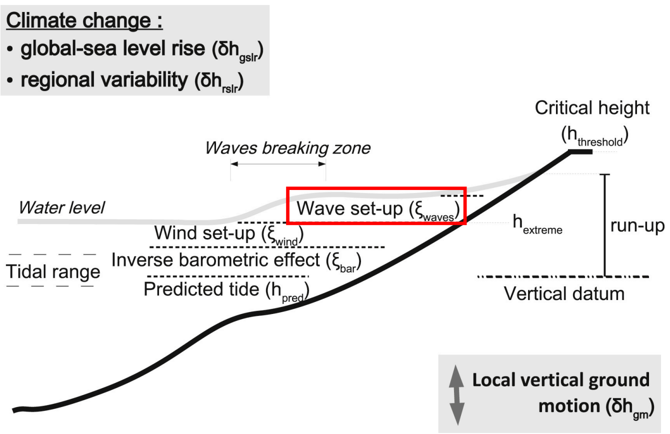
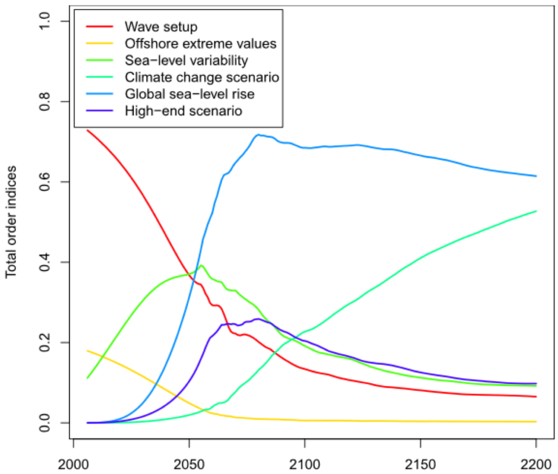
If we use the model for 'what-if' analysis, what are the controls the model (system) output?

Application to coastal flood risk model

Le Cozannet et al *EMS* 2015





So in summary we can use UA/SA to:

_ support model calibration

_ quantify importance of data vs parameter uncertainty > identify priorities for uncertainty reduction

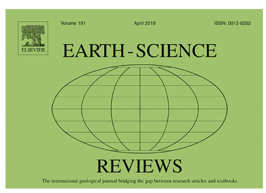
_ test if the model behaves consistently with our expectations ("validation")

_ identify key controls of models/systems

…

What has Global Sensitivity Analysis ever done for us? A systematic review to support scientific advancement and to inform policy-making in earth system modelling

Thorsten Wagener[1,2] and Francesca Pianosi[1,2]
[1]Department of Civil Engineering, University of Bristol, UK
[2]Cabot Institute, University of Bristol, UK



HOW

to do UA/SA?

WORKFLOW FOR UA/SA

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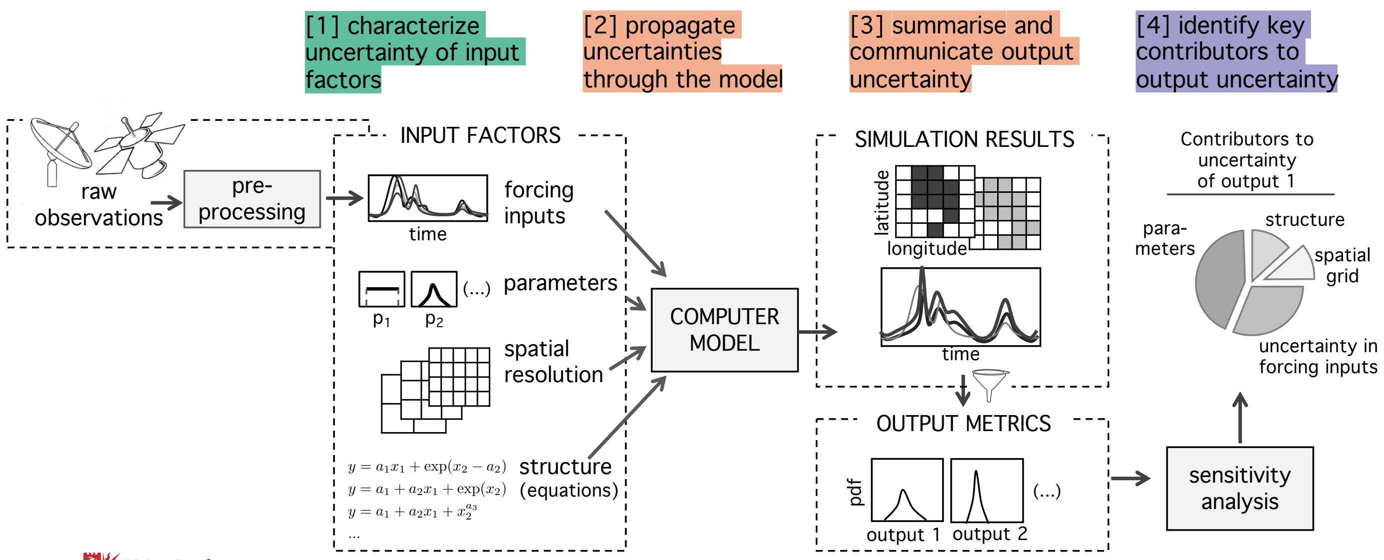
Workflow for UA/SA



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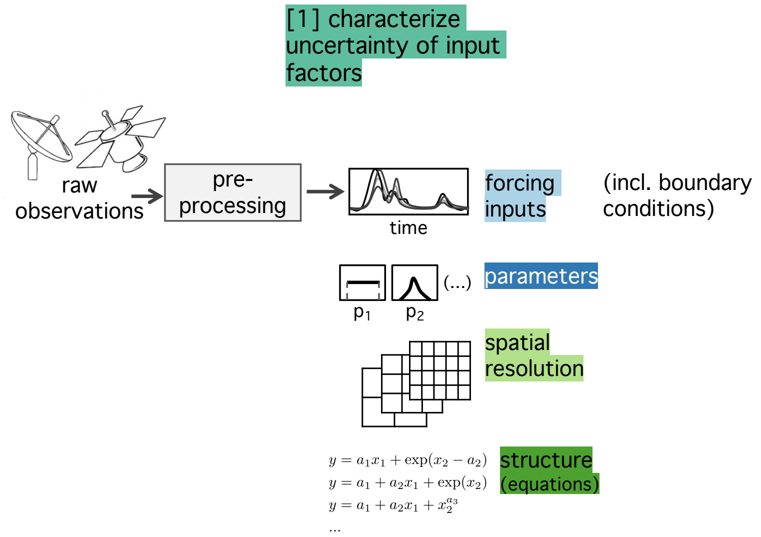
Sources of uncertainty in a model



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Characterizing the sources of uncertainty

_ Probability Distribution Functions (PDFs)
 _ Ranges / list of possible values
  _ Order of magnitude
   _ Sign/direction of change
    _ Governing factors, key indicators and relationships
      (describe pre-conditions that would lead to different values of the uncertain factor)

Increasing uncertainty in uncertainty characterization

[Kadlikar et al CRG 2005]

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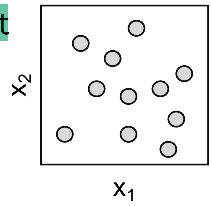
In this lecture we will focus on uncertainties “that can be sampled”

_ Probability Distribution Functions (PDFs)
_ Ranges / list of possible values
_ Order of magnitude
_ Sign/direction of change
_ Governing factors, key indicators and relationships

(describe pre-conditions that would lead to different values of the uncertain factor)

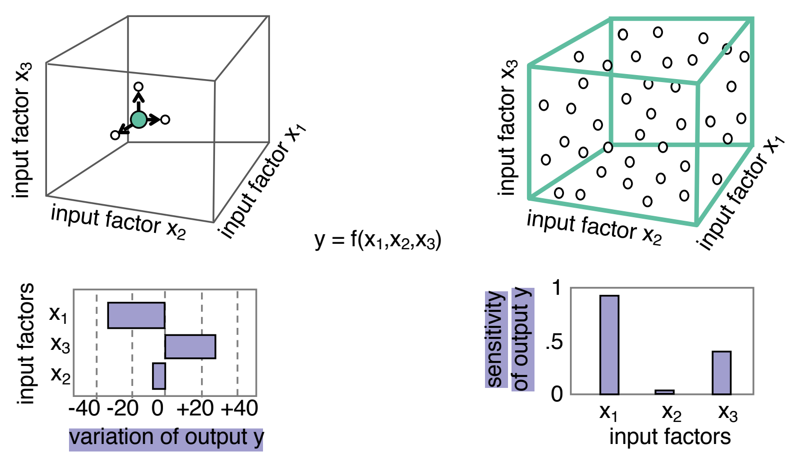
uncertainties that can be sampled



There are two very different approaches: sampling around a baseline, or across the full variability space

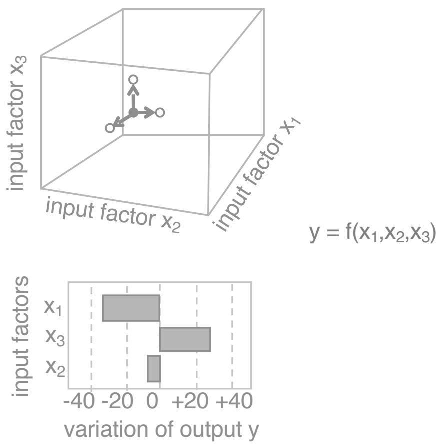
Local SA investigates the effects of variation of uncertain inputs from a baseline point

Global SA investigate the effects of variation of uncertain inputs across their entire variability space

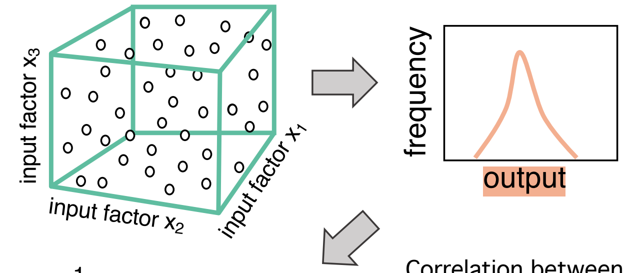


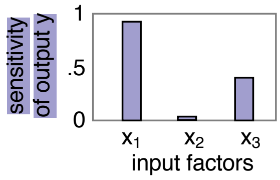
In the ‘global’ approach, there are different ways to define sensitivity indices from the input-output sample

Local SA investigates the effects of variation of uncertain inputs from a baseline point



Global SA investigate the effects of variation of uncertain inputs across their entire variability space





_ Correlation between inputs and output

_ Contribution to variance of output

_ …

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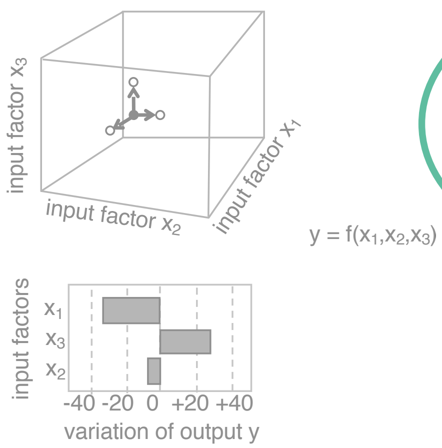
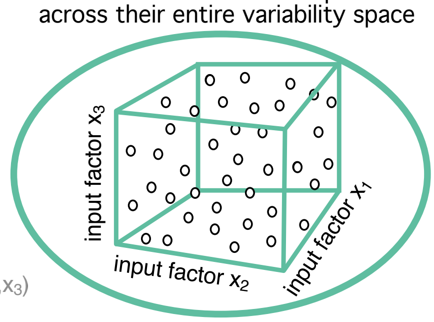
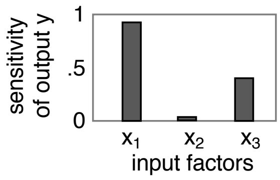
In the ‘global’ approach, some key questions around the sampling approach arise:

Local SA investigates the effects of variation of uncertain inputs from a baseline point



Global SA investigate the effects of variation of uncertain inputs across their entire variability space





a. how do we define the variability space?

b. how many points do we sample? (sample size)

c. how do we place them? (sampling strategy)

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Definition of input variability space: different approaches (and sources of information) can be used, depending on the type of uncertain inputs

Uncertain assumptions/implementation choices:

> discrete description: list of all possible values/choices

Uncertain data and parameters:

> continuous description: probability distribution or ranges *based on*

_ typical errors in data collection and interpolation [McMillan et al *HP* 2012]
_ expert judgements [Morris et al *EMS* 2014]
_ knowledge about the specific catchment

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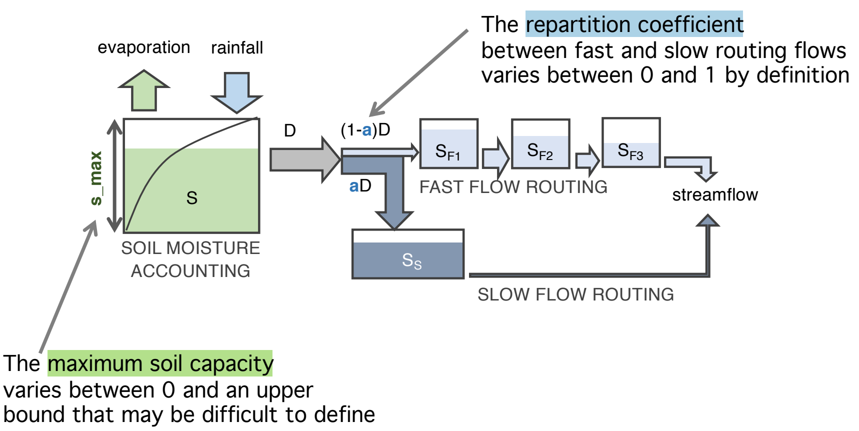
Sometimes the range/distribution of the uncertain inputs is univocally defined by their own meaning, but most often different definitions are possible

Example for a lumped rainfall-runoff model (Hymod)



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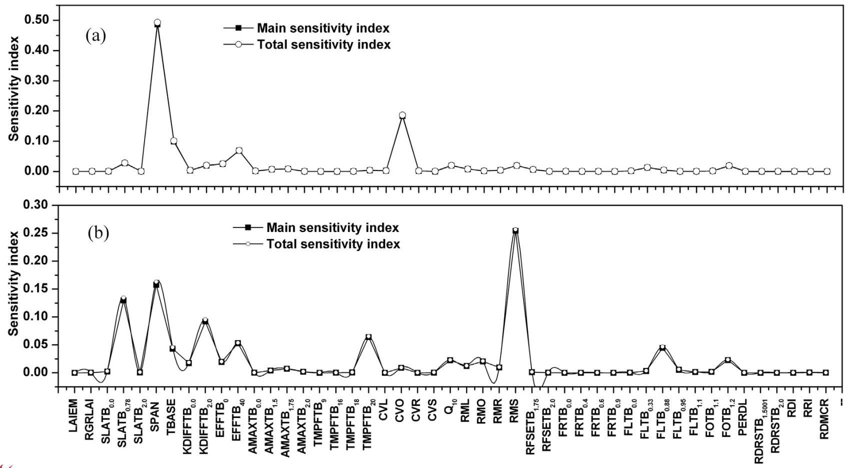
When different definitions of the ranges/distributions are possible, the choice made can significantly condition UA/SA results

Example from SA of a crop growth model



Sensitivity of simulated crop yield based on 'literature' parameter ranges

… based on 'expanded' parameter ranges

[Wang et al. *EMS* 2013]

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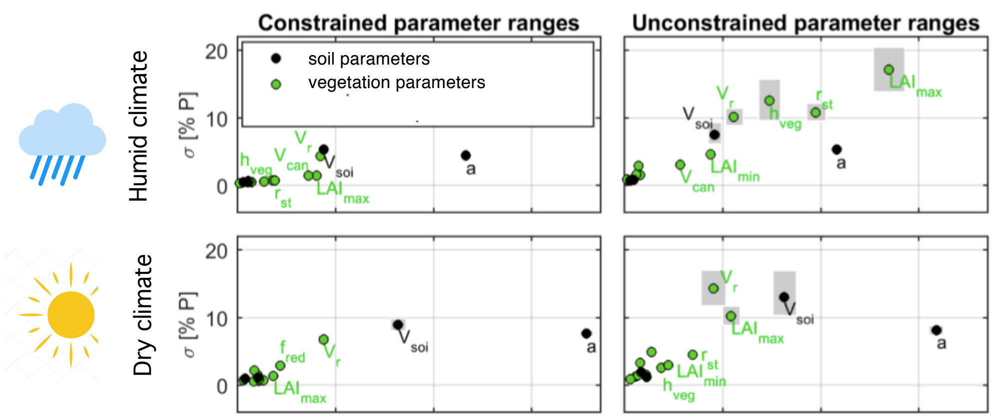
The choice of the ranges also depend on the purpose of the UA/SA (what question is asked)

Example from SA of a karst groundwater recharge model



Sensitivity of simulated recharge to uncertainty about a specific place

Sensitivity of simulated recharge to variability across places

Sarrazin et al *GMD* 2018

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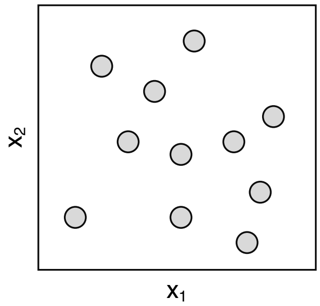
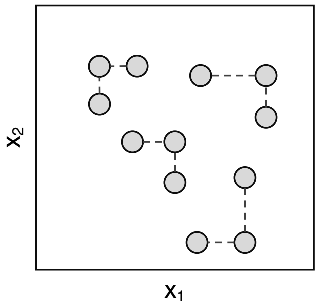
Choice of the sampling strategy: most UA/SA methods use 'generic' sampling strategies but some methods require a 'tailored' strategy

Generic strategies include e.g. Latin Hypercube sampling or Quasi-random sequences



Tailored strategies are e.g. the 'radial' OAT strategy [Campolongo et al. *CPC* 2011] for the Elementary Effect Test



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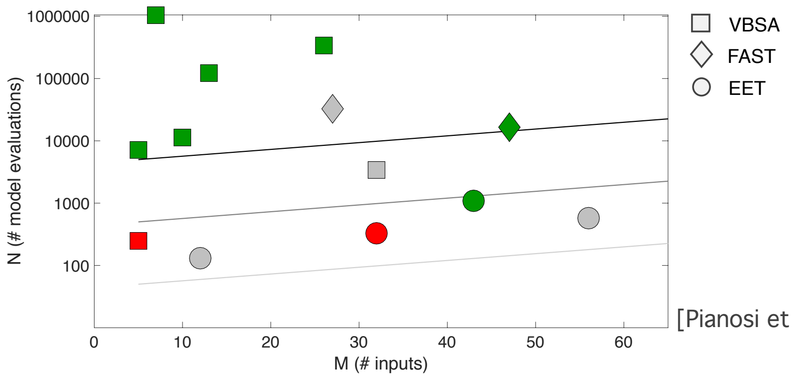
As for the sample size (N), we expect it to increase with the number of uncertain inputs (M). However the proportionality rate varies significantly from one method to another, and from one application of the same method to another



[Pianosi et al. *EMS* 2016]

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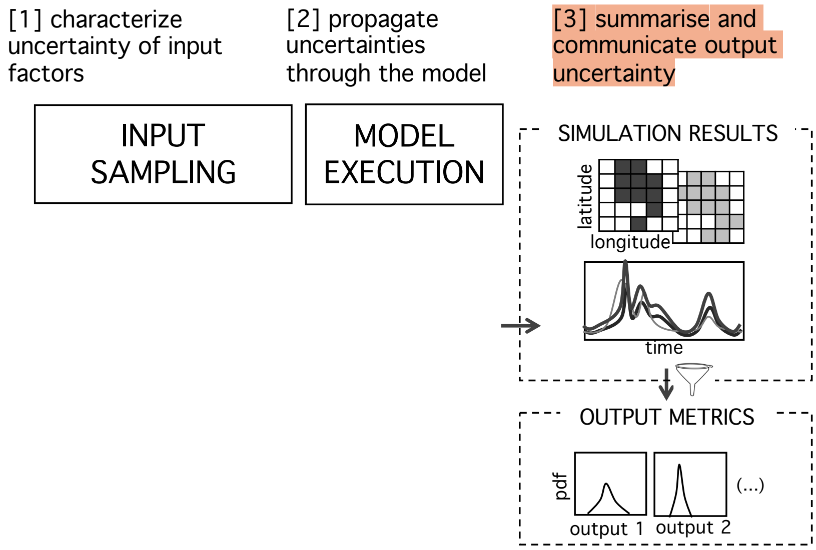
Workflow for UA/SA



Hydrological models typically provide predictions for a range of variables, often distributed over time and space

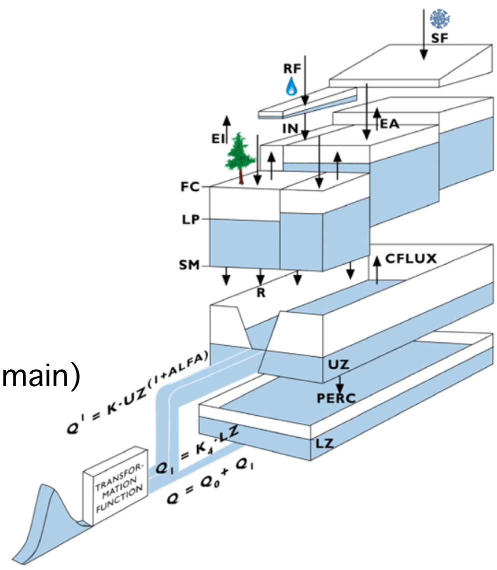
However, GSA is typically applied to a subset of these variables, and often after aggregation over time/space via:

_ a performance metric against observations (e.g. RMSE of streamflows)

or

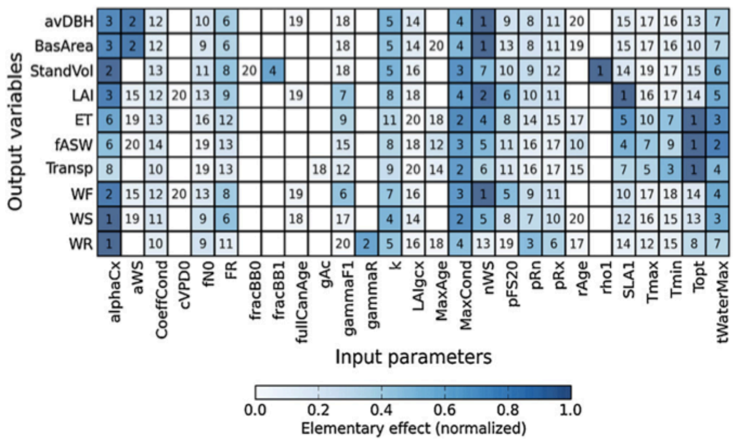
_ a statistic of the model predictions (e.g. maximum predicted streamflow over the domain)

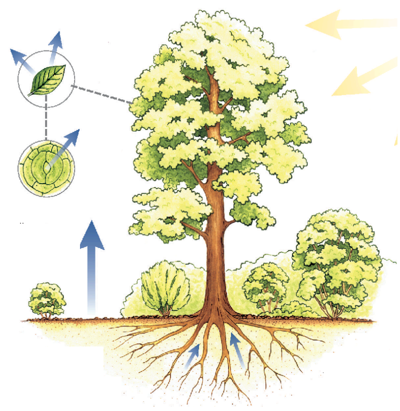
Hence, a key choice in GSA of hydrological models is that of the output metric



Typically, different output metrics will exhibit different sensitivities, hence the choice of which output metrics to consider is crucial

Application to a forest growth model





[Song et al. *EcM* 2012]

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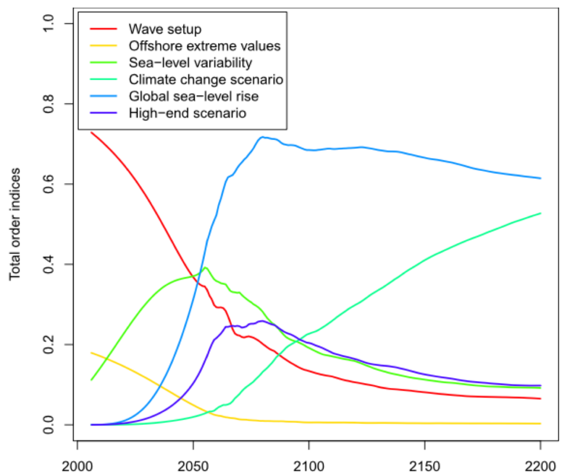
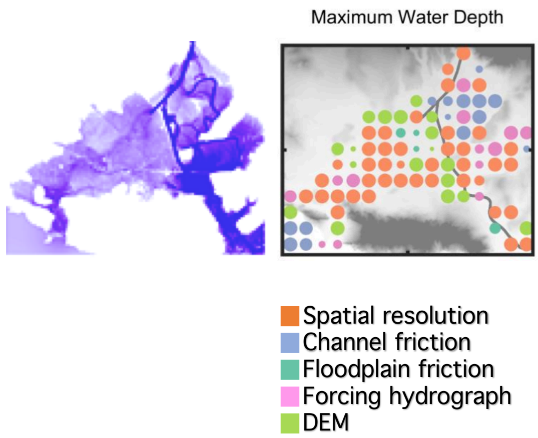
We can also consider spatially or temporally distributed outputs, and derive time series or spatial patterns of sensitivity indices





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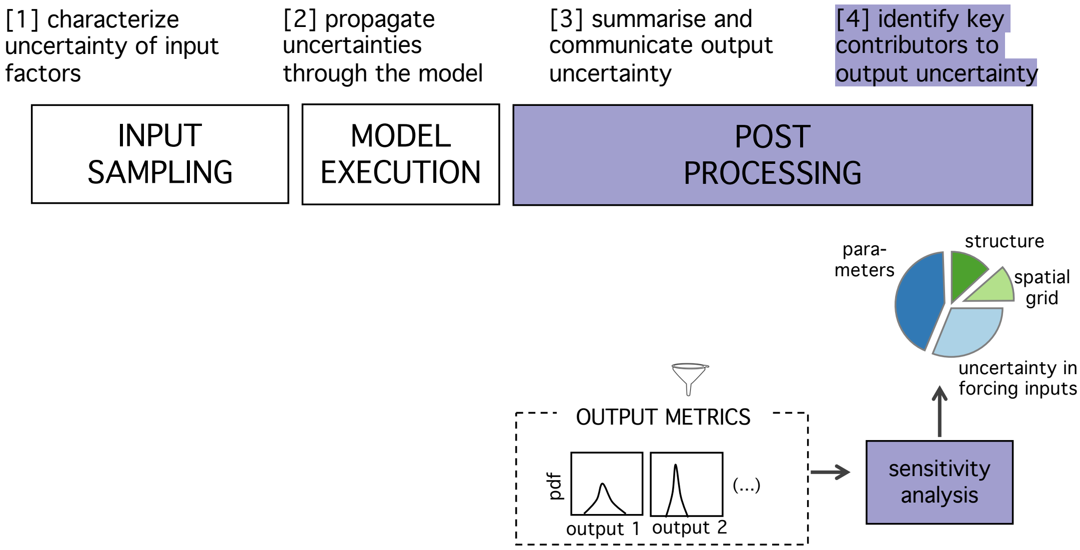
Workflow for UA/SA



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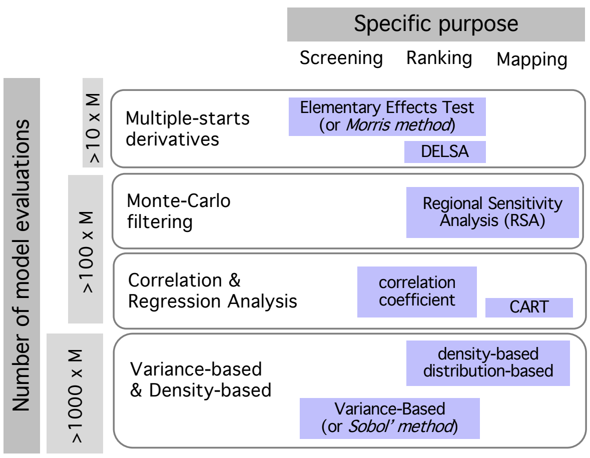
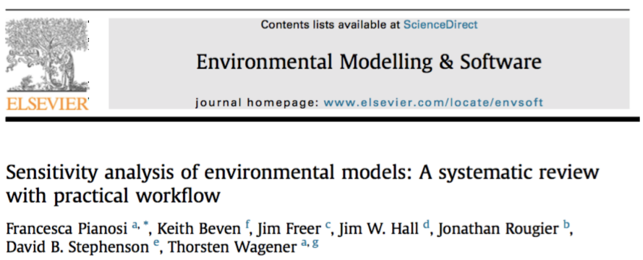
Many methods exist, which rely on different definitions of 'sensitivity' and are more or less suitable for specific problems or purposes



M = number of input factors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ELSEVIER

Sensitivity analysis of environmental models: A systematic review with practical workflow

Francesca Pianosi [a, *], Keith Beven [f], Jim Freer [c], Jim W. Hall [d], Jonathan Rougier [b], David B. Stephenson [e], Thorsten Wagener [a, g]

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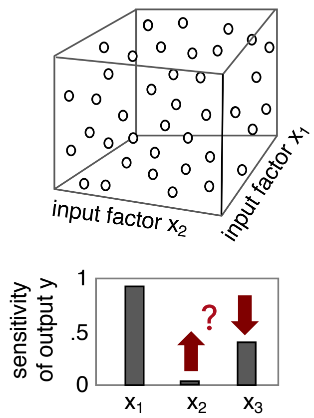
Given that we compute sensitivity indices from a sample of inputs and outputs, our GSA results implicitly depend on the sample we used

An obvious question then is:

how much different would the results be if we used a different sample?



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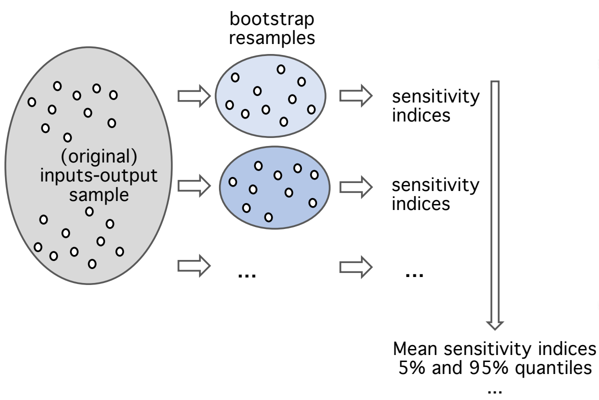
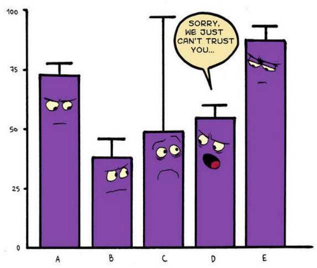
In order to assess the robustness of our sensitivity estimates to the chosen sample, *without re-running the model*, we can use bootstrapping





SOURCE: Reading Local Group of the Royal Statistical Society

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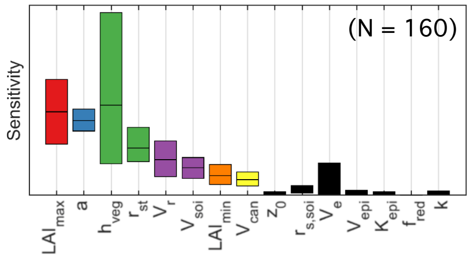
If the confidence intervals of our sensitivity indices are not "small enough", we need to increase the sample size



black line: mean sensitivity index
bar: 90% confidence interval

[Sarazzin et al *EMS* 2016]

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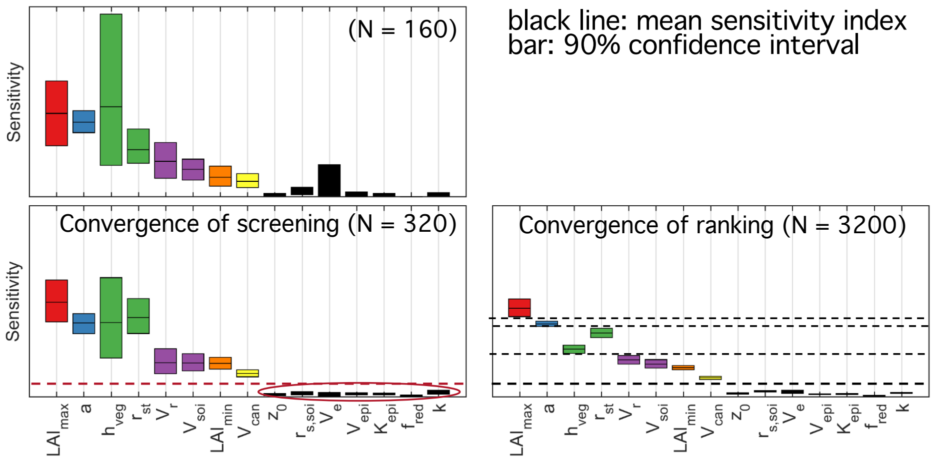
The definition of "small enough" depends on the goal of our GSA



black line: mean sensitivity index
bar: 90% confidence interval

[Sarazzin et al *EMS* 2016]

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CONCLUSIONS

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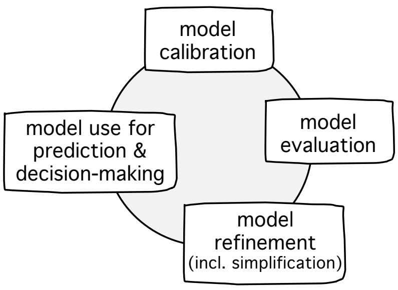
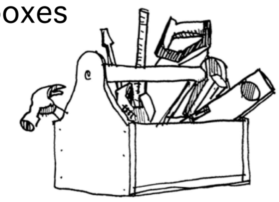
UA & SA are very useful techniques to investigate the propagation of uncertainty through our models and hence support their calibration, improvement, evaluation and use for inference or decision-making

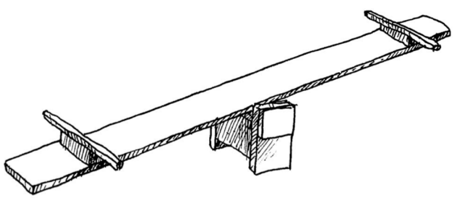


Many methods are available as well as (free and open source) numerical packages and toolboxes



www.safetoolbox.info
(matlab, R and Python)

The key to a successful application often is in making 'good' set-up choices (definition of input variability space, choice of outputs, etc)



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Core References

Pianosi et al. 2015. A Matlab toolbox for Global Sensitivity Analysis. *Environmental Modelling and Software,* 70. (open access: www.sciencedirect.com/science/article/pii/S1364815215001188)

Pianosi et al. 2016. Sensitivity analysis of environmental models: A systematic review with practical workflow. *Environmental Modelling and Software*, 79.
(open access: www.sciencedirect.com/science/article/pii/S1364815216300287)

Sarrazin et al. 2016. Global Sensitivity Analysis of environmental models: Convergence and validation. *Environmental Modelling and Software*, 79.
(open access: www.sciencedirect.com/science/article/pii/S1364815216300251)

Wagener and Pianosi (in press) What has Global Sensitivity Analysis every done for us? A systematic review to support scientific advancement and to inform policy-making in earth system modelling *Earth-Science Reviews* (currently accessible at https://eartharxiv.org/g9ma5/)

Noacco et al. (in press). Matlab/R workflows to assess critical choices in Global Sensitivity Analysis using the SAFE toolbox. *MethodsX* (currently accessible at: https://eartharxiv.org/pu83z/)

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